BICATEGORIES, TWO-DIMENSIONAL COHOMOLOGY, GALOIS COOBJECTS, PSEUDOMONOIDS AND TEH BRAUER GROUP

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ABSTRACT

Classical results state that the Brauer group of a commutative ring, and the group of Galois objects of a cocommutative Hopf algebra can be interpreted as a second cohomology group. The aim of this talk is to present a unifying theory, and also to present an algebraic interpretation of the third cohomology group. This interpretation involves classification of certain monoidal categories, and is related to the theory of quasi-bialgebras. The well-known 7 term long exact sequence of Chase and Rosenberg will be extended to a 10 term exact sequence.
A Picard groupoid - also termed symmetric cat-group - is a symmetric monoidal groupoid $\mathbb{G}$ such that every object I has an inverse I^* in the sense that $I \otimes I^* \cong k$, the unit object of the category. $\mathbb{G}$ is called restricted if $c_{I,I}$ is the identity on $I \otimes I$, for all I. Here c is the symmetry. Picard groupoids can be viewed as the categorical generalization of abelian groups. Given a complex of abelian groups, we can define a sequence of restricted Picard groupoids. The isomorphism classes in each of these groupoids form an abelian group, and these are the cohomology groups of the complex. This is an elementary observation, but the interesting aspect is that this construction can be pushed to a higher dimension. We can define complexes of restricted Picard groupois, and to such a complex, we can associate a sequence of bicategories. Equivalence classes in each of these bicategories form an abelian group, and these are called two-dimensional cohomology groups.
Complexes of restricted Picard groupoids can be obtained from cosimplicial Picard groupoids. In turn, cosimplicial Picard groupoids can be constructed starting from a commutative bialgebroid A over a commutative k-algebra.. The corresponding cohomology is called the Harrison cohomology of the bialgebroid, and the associated bicategories of cocycles are denoted by $\underline{\underline{Z}}^n(A, \underline{\mathrm{Pic}})$.
Classical results about cohomological interpretation of, for instance, the Brauer group, and the group of Galois coobjects over a commutative Hopf algebra, can be refined to 2-equivalences between certain bicategories and the bicategories of cocycles. For example, the Azumaya algebras split by a faithfully projective extension can be organized into a bicategory, and this

bicategory is 2-equivalent to $\underline{\underline{Z}}^2(R\otimes R, \underline{\mathrm{Pic}})$. Passing to equivalence classes, we obtain the well-known theorem that the split part of the Brauer group is isomorphic to the Amitsur cohomology group $H^2(R\otimes R, \underline{\mathrm{Pic}})$.
More generally, have algebraic interpretations of $\underline{\underline{Z}}^n(A, \underline{\mathrm{Pic}})$ for $n = 0, 1, 2, 3$; these are summarized in the following table.

n	$\underline{\underline{Z}}^n(A, \underline{\mathrm{Pic}})$	0-cells	dimension
0	$\mathbb{G}_m(R^{\mathrm{co}A})$	elements	discrete
1	$\underline{\mathrm{Pic}}^A(R)$	invertible A-comodules	category
2	$\underline{\underline{\mathrm{Gal}}}(A)$	Galois coobjects	bicategory
3	$\underline{\underline{\mathrm{PM}}}(A)$	pseudomonoids	bicategory

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