

THE SOLUTION TO SIEGEL'S PROBLEM ON SMALL VOLUME LATTICES

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Abstract:

We outline in very general terms the history and the proof of the identification of the minimal covolume lattice of hyperbolic 3-space as the 3-5-3 Coxeter group extended by the involution preserving the symmetry of this diagram. This gives us the smallest regular tessellation of hyperbolic 3-space. This solves (in three dimensions) the problem posed by Siegel in 1945 (Siegel solved this problem in two dimensions by deriving the Signature formula identifying the $(2,3,7)$ -triangle group as having minimal co-area). There are strong connections with arithmetic hyperbolic geometry in the proof and the result has applications in the maximal symmetry groups of hyperbolic 3-manifolds (in much the same way that Hurwitz 84g-84 theorem and Siegel's result do).