

Some results and problems on unique-maximum colorings of plane graphs

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Abstract

A unique-maximum coloring of a plane graph G is a proper vertex coloring by natural numbers such that each face α of G satisfies the property: the maximal color that appears on α , appears precisely on one vertex of α (or shortly, the maximal color on every face is unique on that face). Fabrici and Göring proved that six colors are enough for any plane graph and conjectured that four colors suffice. Thus, this conjecture is a strengthening of the Four Color Theorem. Wendland later decreased the upper bound from six to five.

We first show that the conjecture holds for various subclasses of planar graphs but then we disprove it for planar graphs in general. Thus, the facial unique-maximum chromatic number of the sphere is not four but five. In the second part of the talk, we will consider various new directions and open problems.

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