



UNITED ARAB EMIRATES UNIVERSITY



# **2023 UAEU-AUA International Workshop on Delay Differential Equations Applications to Immunology and Infectious Diseases (DDEs-AIIDs)**

from 23–24 Nov., 2023, UNITED ARAB EMIRATES UNIVERSITY

College of Science  
Department of Mathematical Sciences  
UAE  
23–24 November, 2023

## **Book of Abstracts**



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## 1 Organizing Committee

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## 2 Workshop Overview

A growing interest has been shown in delay differential equations (DDEs) due to their important applications in biology, medicine, immunology, and epidemiology. Using mathematical models and DDEs of infectious diseases, we can study the pathogenesis of disease processes, predict outbreaks, and minimize outbreak severity and frequency. This focused workshop DDEs-AIID will bring together mathematicians and life science scholars at AUA universities, as well as outside of AUA universities, to meet, share, and discuss research interested in mathematical modeling with delay differential equations and their applications to immunology and infectious diseases. There will be a round table discussion on open problems and questions in the context of infectious diseases, cancer, and chronic diseases, as well as future directions in multidisciplinary research. Potential topics include but are not limited to the following i) DDEs mathematical analysis and biological interpretation, ii) Models of host-pathogen interactions, iii) Models of cancer immune response, iv) Models of immunological regulation, v) Therapy and vaccination models, vi) Models of multiscale immunoepidemiology, vii) Optimal control with therapy, viii) Immune response, signaling pathways, and regulatory networks

The scientific program of the workshop consists of 50-minute plenary lectures, 30-minute invited lectures and 20-minute contributed talks.

The selected papers presented will be published in the Special Issue "Delay Differential Equations and Applications to Immunology and Infectious Diseases" of "Mathematical Biosciences and Engineering", published by AIMS.

## 3 Keynote Speakers

1. Prof. Gennady Bocharov, Marchuk Institute of Numerical Mathematics, Russian Academy of Sciences, Moscow, Russia;
2. Prof. Andreas Meyerhans, Infection Biology Laboratory Department of Medicine and Life Sciences, Universitat Pompeu Fabra, Barcelona, Spain;
3. Prof. Vitaly Volpert, Institut Camille Jordan, UMR 5208 CNRS, University Lyon 1, 69622 Villeurbanne, France.

## 4 Invited Speakers

1. Prof. Radouane, Yafia, Morocco: Ibn Tofail University
2. Prof. Abdulla Azamov, Uzbekistan: Institute of Mathematics
3. Dr. Ardak KASHKYNBAYEV, Kazakhstan: Nazarbayev University
4. Prof. Jiaxu Li, USA: Department of Mathematics, University of Louisville

### **Acknowledgement**

We are grateful for the support of Department of Mathematical Sciences (UAEU), Asian Universities Alliance (AUA) for support and helping make this workshop possible. . . . Fathalla A. Rihan

## 5 Abstracts

# Understanding the pathogenesis of virus infections using mathematical models based on delay differential equations

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### Abstract

Mathematical immunology is a rapidly developing highly demanded field of applied mathematics. It aims to bring the rigor of mathematics and the power of computer technologies to describe, analyze and predict the dynamic complexities of the immune system. The immune system functions to maintain the antigenic homeostasis of the host organism. Virus infections induce a set of antiviral immune responses with the outcome depending on the dynamics of the pathogen-host interaction. We review the structure of representative mathematical models describing processes of multi-physics nature formulated with delay differential equations. We apply the models to studying the pathogenesis of infections such as SARS-CoV-2, HIV-1, HBV and to quantifying the growth kinetic parameters of CFSE labelled lymphocytes. To this end, we apply a range of analytical methods including the sensitivity-, bifurcation-, and dynamic uncertainty analyses. The models provide core building blocks for developing a novel class of mathematical models and methods for implementation of a multi-physics description and analysis of the functioning of immune system under normal conditions and in the course of infectious diseases.

The study is supported by the Russian Science Foundation (grant number 23-11-00116).

# Pathogenic virus infections: on fate regulation and functional cure strategies

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## Abstract

The fates of pathogenic virus infections are determined by the dynamic interplay between the expanding virus and the concomitantly induced immune response in the infected host. Depending on the temporal virus-host relationship, infections are either acute or chronic. In humans, acute infections are usually resolved within few weeks. In contrast, chronic infections are not resolved and, instead, develop when innate and adaptive immune responses are not sufficient to eliminate the invading virus during the primary infection phase. This presentation will address 2 fundamental questions of virus infections: (1) Which immunological features of the infected host organism determine the acute or chronic outcome of a virus infection? (2) Can chronic persistent virus infections be functionally cured by immunotherapeutic interventions? Experiments using the acute and chronic lymphocytic choriomeningitis virus (LCMV) infection model of mice will be presented and discussed. They provide clues about immune system perturbation and functioning that may be of general importance also for other infections like those with the human immunodeficiency virus (HIV).

This work is supported by grants from the Spanish Ministry of Science and Innovation grant No. PRPPID2022-141395OB-I00, la Caixa Foundation (HR17-00199), the Unidad de Excelencia Mara de Maeztu, funded by the AEI (CEX2018-000792-M), and the Russian Science Foundation (grant number 23-11-00116).

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# Mathematical modelling of respiratory viral infections

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## Abstract

In this lecture we will present an overview of recent works on mathematical modelling of respiratory viral infections. We will begin with the investigation of infection progression in cell cultures and in tissues of human body. We will determine viral load and infection spreading speed and we will apply these results to evaluate infectivity and severity of symptoms for different variants of the SARS-CoV-2 infection. Next, will take into account mucus motion in the lung airways and will show how viral infection can lead to mucus accumulation and airway obstruction leading to a life-threatening disease progression.

# Qualitative and Quantitative Features of Delay Differential Equations and Applications to Immunology

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## Abstract

Recently much attention has been given to mathematical modeling of real-life phenomena using differential equations with memory, such as delay differential equations (DDEs). This is due to the fact that introduction memory terms in a differential model significantly increases the complexity of the model. Such a class of DDEs is widely used for analysis and predictions in various areas of life sciences and modern topics in population dynamics, computer science, epidemiology, immunology, physiology, and neural networks. In this talk, we provide a wide range of delay differential models that have a richer mathematical framework (compared with ODEs) for the analysis of biosystems. Qualitative and quantitative features of DDEs are discussed. Some numerical simulations are also provided with tumour-immune interactions to show the effectiveness of the theoretical results.

## References

- [1] Rihan F.A., *Delay differential equations and applications to biology*, Springer (2021).



# Dynamics of A Delayed Spatiotemporal HBV Infection Model with Capsids, CTL Immune Response and General Incidence Function

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## Abstract

This study explores the integrated effects of diffusion and CTL immune response activation delay on the in vivo dynamics of a generalized hepatitis B virus (HBV) infection model in terms of both mathematical analysis and numerical simulations. The considered model involves five compartments such as uninfected and infected hepatocytes, HBV DNA-containing capsids, viruses, and CTL cells. We model the infection process by a general incidence function with some biologically feasible assumptions and, as a result, it includes a wide variety of popular incidence functions utilized in the existing literature. We then establish the well-posedness of the considered model in terms of the existence, uniqueness, non-negativity, and boundedness of the solutions. Further, three different steady states such as disease-free, immune-free and infection steady states are identified for the model under threshold conditions depending upon the reproduction numbers  $\mathcal{R}_i$  and  $\mathcal{R}_{CTL}$ . With the help of Lyapunov functions and LaSalle's invariance principle, we prove that both the disease-free and immune-free steady states are globally asymptotically stable under suitable conditions. On the other hand, infection steady state with CTL immune response exhibits the Hopf bifurcation with respect to the delay parameter. Finally, some corroborative numerical simulations are presented with a particular choice of incidence function.

# Applications of Delay Differential Equations in Medicine

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## Abstract

**Motivation and Aims:** Recently much attention has been given to mathematical modeling of real-life phenomena using differential equations with memory, such as delay differential equations (DDEs). This is due to the fact that introduction memory terms in a differential model significantly increases the complexity of the model. Such a class of DDEs is widely used for analysis and predictions in various areas of life sciences and modern topics in population dynamics, computer science, epidemiology, immunology, physiology, and neural networks. In this paper, we provide a wide range of delay differential models with a richer mathematical framework (compared with ODEs) for analyzing biosystems. Qualitative and quantitative features of DDEs are discussed. Some numerical simulations are also provided to show the effectiveness of the theoretical results.

**Results and Conclusions:** 1. DDEs of real-life phenomena have potentially more interesting dynamics than equations without memory; 2. The time delay (memory) is incorporated into systems to describe resource regeneration times, maturation periods, reaction times, feeding times, gestation periods, etc.; 3. Studying the qualitative behaviour of DDEs is essential for ensuring safe applications or real problems; 4. The presence of memory in the model enriches the dynamics of the model: It can stabilize or destabilize the system. 5. Mono-implicit continuous RK schemes are stable and suitable for stiff and nonstiff DDEs. 6. Sensitivity functions clearly demonstrate the measure of the importance of the input parameters. 7. Sensitivity functions enable one to assess the relevant time intervals for the identification of specific parameters.

## References

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# Global behavior in differential delay models of population dynamics and physiology

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## Abstract

Dynamics of solutions in simple form scalar delay differential equations are studied. The first class includes the Mackey-Glass type equations modeling several distinct processes such as population fluctuations, mechanisms of blood cell production, some others [2, 3, 4]. The second equation is recently proposed as a mathematical model of megakaryopoiesis (platelet production) [1]. Sufficient conditions are derived when the unique positive equilibrium is globally asymptotically stable. The stability conditions are given in terms of induced interval maps, one set being delay independent criteria and another one involving the delay. The existence of periodic solutions slowly oscillating about the equilibrium is also established. The periodic oscillations in the models exist when the positive equilibrium is linearly unstable. The proof of existence can be done by several methods, in particular by following the established ejective fixed point techniques with necessary modifications due to the specific form of the equations.

## References

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# Crossover Dynamics of Hybrid Fractional Order Monkeypox Disease Model with Time Delay: Numerical Simulations

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## Abstract

Monkeypox is a rare viral zoonic illness caused by the monkeypox virus. It was first, identified in 1958. The virus-causing monkeypox comes from the same family of viruses as the variola virus, which causes smallpox. It was first reported in central Africa in 1970. Its symptoms are fever, rash, and lymphadenopathy. In extreme cases, carriers can have pneumonitis, encephalitis, sight-threatening keratitis, and secondary bacterial infections. Children, pregnant women, and immunocompromised persons may be at risk of more severe disease with monkeypox. In this talk, a mathematical model of monkeypox disease is extended using the fractional stochastic-hybrid fractional order deterministic piecewise derivatives. A parameter  $\xi$  is presented in order to be consistent with the physical model problem. The positivity, boundedness, and existence of the solutions for the proposed model are discussed. New numerical algorithms are improved to study the proposed model. These methods are the nonstandard Caputo proportional constant Adams-Bashfourth sixth step method and the nonstandard Euler Maruyama technique to solve the fractional stochastic model. Several numerical tests demonstrate the efficiency of the method and support the theoretical results. The new data simulation and various solution approaches are the novelty of this research. This work aims to explore the transmission pattern of a monkeypox outbreak in the United States.

# Deterministic and stochastic SIRC epidemic model with time-delay for COVID-19

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## Abstract

An epidemic model based on deterministic and stochastic SIRC has been proposed for COVID-19, taking into account cross-immune classes and time delays in transmission terms. Under some criteria, infection-free and endemic steady states are asymptotically stable. Using the next-generation matrix method, the reproduction number  $\mathcal{R}_0$  for the deterministic model is calculated according to contact rate, recovery rate, and other parameters. Positive global solutions exist and are unique for stochastic analysis. Stochastic models have a smaller reproduction number than deterministic models of the same type  $\mathbb{R}_0$ . We obtain conditions for the extinction of the disease by using the stochastic Lyapunov functional. It appears that white noise and time delay play a significant role in controlling the spread of the virus.

## References

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# Exploring the Influence of Fear on the Progression of an Infectious Disease in a Delayed Spatiotemporal Epidemic Model Featuring Both Local and Nonlocal Dispersion

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## Abstract

The paper investigates an epidemic model known as the Susceptible-Protected-Infected-Recovered (SPIR). This model incorporates a mixed diffusion framework, comprising both local and nonlocal diffusions, to capture the influence of fear among the population when it comes to infection transmission. We establish the mathematical well-posedness of this model, which entails demonstrating the existence, positivity, and uniqueness of its solution. One key aspect of our analysis involves deriving a variational expression for the basic reproduction number, denoted as  $\mathfrak{R}_0$ . This parameter serves as a crucial threshold for the epidemic dynamics. Specifically, when  $\mathfrak{R}_0$  is less than 1, we establish that the epidemic will eventually die out, signifying the global asymptotic stability of the infection-free equilibrium state. However, when  $\mathfrak{R}_0$  exceeds 1, we demonstrate that the solution will persist uniformly, and an endemic equilibrium state emerges. To ascertain the global stability of this endemic equilibrium, we employ Lyapunov function techniques. We consider two distinct scenarios: one where the diffusion coefficient for susceptible populations is set to zero, and another where the diffusion coefficient for infected populations is set to zero. Furthermore, we conduct a comparative analysis with the classical SIR epidemic model to determine the necessary protective measures required to bring the disease under control. This involves reducing  $\mathfrak{R}_0$  below one, which can be achieved through the implementation of appropriate protection strategies.

## Time Lag Effect of Prey Predator Model with Allee Effect

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### Abstract

We have studied a predator-prey system with time lag impact for intra-specific interaction and a substantial Allee effect on prey population growth. We examined the stability of coexisting equilibrium points using the discrete time delay as a bifurcation parameter and showed the existence Hopf-bifurcation when the discrete delay crosses a lumping parameter value. The normal form technique and centre manifold theory have been used to study the aspect of direction and stability of the periodic Hopf-bifurcating solution. All the analytical results are compared with numerical simulations for biological significance.

**Keyword:** Allee-effect, Stability, Hopf-bifurcation, Intra-specific competition.

# A fractional-order delay differential tumor-immune model: external treatment and optimal control strategies

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## Abstract

In the context of external treatment strategies, this paper aims to present a mathematical model of the tumor-immune system. The governing equations of the model are fractional-order delay differential equations, which incorporate a control variable. The inhibition of transforming growth factor beta by itself to provide restricted clinical benefits. However, when given combined with external treatments, it has the potential to significantly enhance the anti-tumor response of the immune system. The purpose of this study is to employ optimal control theory in order to minimize the cost associated with external treatment strategies and decrease the burden of tumor cells. It has been shown that the model's solutions are non-negative and that the existence of an optimal control. We solve the state system forward in time and the adjoint system backward in time to get a numerical approximation of the optimal control problem. Numerical simulations demonstrate that combination of the transforming growth factor beta and external treatments can significantly reduce the tumor load within a short time frame.



# Robust exponential $H_\infty$ filtering for discrete-time complex-valued neural networks with time-varying delays

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## Abstract

This paper aims to design a compatible filter for a class of delayed discrete-time complex-valued neural networks (CVNNs) having uncertain weighting parameters subject to the  $H_\infty$  performance measure. For this notion, the complex-valued filter scheme is designed for the proposed uncertain CVNNs with regard to the available output measurements. Following the idea of weighted summation inequalities (WSIs) in [1] and complex-valued reciprocal convex matrix inequality [2], we have improved the new complex-valued WSIs without requiring any extra decision variable to bound the summation terms included in Lyapunov-Krasovskii functional (LKF). By utilizing these newly proposed complex-valued WSI and prescribed LKF, an attempt has been made to design the robust  $H_\infty$  filter such that the filtering error system attains exponential stability with the appropriate filtering gain matrices. Finally, a numerical example is performed to acknowledge the significance and efficiency of the proposed filtering scheme.

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# Optimal Control of Stochastic Michaelis-Menten Kinetics Based Tumour-Immune Interaction with Time Delay

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## Abstract

Herein, we study an optimal control problem of stochastic model to describe the dynamics of tumour-immune interactions, where the interaction of tumour cells with the immune system is described as Michaelis-Menten. The model includes constant time delay to justify the time needed by the immune system to develop a suitable response after recognizing [1]. We first discuss stability and Hopf bifurcation of the deterministic system [2]. We then explore stochastic stability, and the dynamics of the system in view of environmental fluctuations. Criteria for persistence and sustainability are discussed. Using multiple Lyapunov functions, some sufficient criteria for tumour cell persistence and extinction are obtained. By applying optimal control theory, we seek to optimize the objective function in a random environment and to reduce load of of tumour cells [3]. Optimality conditions and characterization of the control are also discussed. We numerically approximate the solution of the optimal control problem by solving the state system forward and adjoint system backward in time

## References

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# Turing Bifurcation Induced by Cross-Diffusion and Amplitude Equation in Oncolytic Therapeutic Model

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## Abstract

In this paper, we propose a reaction-diffusion mathematical model augmented with self/cross-diffusion in 2D domain which describes the oncolytic virotherapy treatment of a tumor with its growth following the logistic law. The tumor cells is divided into uninfected and infected cells and the virus transmission is supposed to be in a direct mode (from cell to cell). In the absence of cross-diffusion, we establish well posedness of the problem, nonnegativity and boundedness of solutions, non-existence of positive solutions, local and global stability of the nontrivial steady state and the non-occurrence of Turing instability. In the presence of cross-diffusion, we prove the occurrence of Turing instability by using the cross-diffusion coefficient of infected cells as a parameter. To have an idea about different patterns, we derive the corresponding amplitude equation by using the nonlinear analysis theory. In the end, we perform some numerical simulations to illustrate the obtained theoretical results.

## References

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# Mathematical Model Integrating Vaccine and Variable Immunity Period in Infectious Disease Dynamics

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## Abstract

We formulate a mathematical model for an infectious disease incorporating a vaccine and variable immunity duration. Our model integrates the behaviors of susceptible, infected, and recovered individuals, along with the impacts of vaccination and the waning of immunity. We establish a formula for the fundamental reproductive rate and employ it to explore the local asymptotic stability of both the disease-free equilibrium (DFE) and the endemic equilibrium (EE). Our findings reveal that the basic reproduction number is highly sensitive to several factors, most notably the rate of immunity loss. Through time series analysis, we evaluate the models outcomes both with and without vaccination, underscoring the vaccines significance in disease management. Furthermore, we estimate time series outcomes for both constant and variable immunity durations, yielding divergent results. Our conclusions underscore not only the necessity of vaccination but also the critical importance of the vaccines conferred immunity period.

# Improved lung model for predicting aerosol deposition in lung airways

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## Abstract

A computer model, for predicting deposition of therapeutic or pollutant aerosol particles in the human lung, has been improved to calculate deposition in each airway generation during inhalation, exhalation, and the breath-holding pause. Deposition in an airway is still based on aerosol flow between characteristic curves in the solution domain but the calculation can be simplified as the product of local flow rate and time interval has been found to be independent of airway generation number though the separate components are not. The model is versatile as it can handle continuous aerosols or pulsed ones intended to target specific regions where lung disease may exist. Electrostatic deposition mechanisms may be considered as well as conventional ones: sedimentation, diffusion, and impaction, which can be made operative as required. Functional residual capacity and head volume are input parameters as are particle diameter, charge, density, and initial number concentration. Inhalation and exhalation flow rates and durations, as well as the breath-holding pause, may be adjusted independently to maximize control over the deposition site and level.

# Dynamical analysis of an HIV infection model including quiescent cells and immune response

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## Abstract

This paper presents a comprehensive analysis of an HIV infection model that incorporates quiescent cells and immune response dynamics within the host. The model, represented by a system of ordinary differential equations, captures the intricate interplay between the hosts immune response and the viral infection. The study investigates the fundamental properties of the model, including equilibrium analysis, the computation of the basic reproduction number  $\mathcal{R}_0$ , stability analysis, bifurcation phenomena, numerical simulations, and sensitivity analysis. An infection equilibrium, which reflects the persistence of the infection, and a disease-free equilibrium, which represents the possibility of disease control, are both revealed by the analysis. By applying matrix-theoretical methods, stability analysis confirmed that the disease-free equilibrium is both locally and globally stable for  $\mathcal{R}_0 < \infty$ . The research also reveals a transcritical forward-type bifurcation at  $\mathcal{R}_0 = \infty$ , which denotes a critical threshold that affects the behavior of the system. The temporal dynamics of the model are investigated through numerical simulations, and sensitivity analysis determines the most important variables by examining the effects of parameter changes on the systems behavior.

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# Fractional ordered model for cell level viral transmission dynamics with adaptive immunity

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## Abstract

In this paper we proposed fractional ordered model of cell level pathogen transmission dynamics by considering adaptive immunity. Infected and latently infected cells are considered in the model. The model assumed as infection occur from virus to health cell and from infected cell to health cells. The qualitative behavior of the model is analyzed. From the qualitative analysis basic threshold parameters that affect clearance and maintenance of infection are determined. Effect memory as well as some parameters on the expansion of the infection is done in numerical simulations.

**Keywords:** Fractional Order; Modeling; Cell level; Adaptive immunity

# The SEIR model With Unreported Cases and Markovian Switching

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## Abstract

The study of infectious diseases in humans has become increasingly important in public health. This paper extends the SEIR model to include unreported cases (U), environmental white noise, and the impact of an exogenous event. Environment variation is used to conduct dynamic analysis. We examine the process of switching from one regime to another at random. The ergodicity and stationary distribution criteria are discussed. Using a Lyapunov function, we write down some sufficient conditions for disease extinction. With different intensities of stochastic noises, we calculate the threshold of extinction for the stochastic epidemic system. In order to control the spread of disease, stochastic noise plays an important role. A numerical simulation and a fit to real data have shown that the model and theoretical results are valid.



# Mathematical modeling of the effect of quarantine rate on controlling the infection of COVID19 in the population of Saudi Arabia

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## Abstract

With the development of communications and transportation worldwide, the challenge of controlling epidemiological diseases becomes higher. The COVID19 has put all nations in a lethal confront with a severe disease that needed serious and painful actions. The sooner the actions, the less destructive the impact. In this paper, we incorporate what we believe is crucial but applicable to control the spread of COVID19 in the populations, that is, quarantine. We keep the model as simple as SI Kermack-McKendrick model with an additional compartment of quarantined patients. We established the systems basic properties and studied the stability of the disease-free equilibrium and its relation to the basic reproduction number  $\mathcal{R}$ , in which we calculated its formula. The focus of our study is to measure the effect of quarantine rate on controlling the spread of COVID19. We use the data collected from the Ministry of Health in Saudi Arabia. We studied three different values of the quarantine rate where newly infectious patients are detected and isolated within 14, 7, and 5 days. The simulations show a significant effect of the quarantine where COVID19 can be fully controlled if the newly infected patient enters the quarantine within five days. These results were proposed to the Public Health Authority in Saudi Arabia and approved by the Ministry of Health in which they applied promptly.

# Optimal control for cancer-tumor-immune systems using reinforcement learning

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## Abstract

In this work, a class of cancer-tumor-immune systems with drug dosage acting as control is described as [1, 2]

$$\begin{aligned}\dot{T} &= \mu_c T F(T) - \gamma T I - \kappa_T T u \\ \dot{I} &= \mu_I (T - \beta T^2) I - \delta I + \alpha + \kappa_I I v\end{aligned}$$

In the above set of equations, the values  $T$  represents the tumor volume and  $I$  denotes the immune competent cell densities. The ability of the immune cells to eliminate the tumor cells are denoted through the tumor-cell-elimination rate  $\gamma \in \mathbb{R}$ . The positive constants  $\alpha, \delta, \beta, \mu_I$  denotes the T-cells generation rate, the natural death rate of immune cells and calibrations these interactions and the way cancer cells influence the stimulation of immune cells, respectively. The control input  $u$  determined though drug dosage eliminates the tumor cells. is utilized to denote the booster drug for the immune system. The positive coefficients  $\kappa_T$  and  $\kappa_I$  represent the control capability of the control activity of drug dosage and immune system booster drug, respectively. An optimal control for cancer-tumor-immune systems is discussed using reinforcement learning

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## Hepatitis B epidemic model: analysis and control

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### Abstract

The infection of hepatitis B produces inflammation of the liver and can cause chronic and acute diseases, while it is life-threatening and a major health problem around the globe. The control of this infection is a challenging task because of many reasons such as proper medication, variation of human behavior, vaccination, and particularly the existence of a large number of carriers, however, understanding the temporal dynamics of the infection helps to present appropriate control mechanisms. Thus, a complex dynamical system is needed to find the stability conditions and propose various intervention strategies to forecast the control of the contagious infection of hepatitis B. We develop a model to investigate the temporal dynamics and suggest a control mechanism for the infection of hepatitis B. The well-posedness will be shown to prove the feasibility of the proposed epidemic problem and find the threshold parameter to analyze the model equilibria and its stability. We also discuss the sensitive analysis of the reproductive number to quantify the parameters that are more sensitive to disease spreading and control. Based on the model dynamics and sensitivity, we investigate effective procedures to minimize the infection of hepatitis B and develop algorithms to verify the theoretical work with the help of numerical experiments.

# Comprehensive cost-effectiveness analysis of a new compartmental model for bacterial meningitis considering the influence of the media

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## Abstract

Mathematical epidemiology has paid some attention to the study of bacterial meningitis because of the severity of the disease and the way it spreads through a population. In this paper, we formulated a new compartmental model for meningitis to study the impact of media on the reduction of the severity of the disease in Ghana. We obtain the control reproduction number and the herd immunity rate in the presence of vaccination. We noticed that the meningitis death rate can be control with an increase in publicity of the dynamics of the spread of the disease and the ability to seek immediate medical treatment. We conduct a sensitivity analysis using Latin hypercube sampling. It is noticed that the rate of transmission and that of, vaccination and the media have a negative nonlinear correlation. The effects of media, vaccination, and treatment as a function of time are also investigated using an optimal control model. We display the efficiencies of the controls and demonstrate the mean and incremental cost-effectiveness ratios. The cost of controlling meningitis in the presence of media, vaccination, and treatment is discussed, and we hope that this study will help to further educate the public on this topic.

**Keywords:** Control Reproduction Number, Media, Vaccination, Treatment, Cost-effectiveness analysis

# Dynamics of HIV-COVID-19 Co-Infection: A Fractional Order Model and Analysis using Laplace-Adomian Decomposition Method

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## Abstract

The co-infection of HIV and COVID-19 presents a significant health challenge with potentially severe consequences. Understanding the dynamics of HIV-COVID-19 co-infection is crucial for developing effective control measures and optimizing healthcare strategies. In this study, we develop a mathematical model based on Caputo fractional order differential equations to capture the complex dynamics of co-infection. The model incorporates the transmission dynamics of HIV and COVID-19, the immune response of the host, and the effects of treatment interventions. To solve the fractional order model, we employ the Laplace Adomian decomposition method, a powerful technique for obtaining approximate solutions of fractional order differential equations. By simulating the interactions between these factors, we gain insights into the spread of coinfection and identify critical factors contributing to its progression. Additionally, we analyze the convergence properties of the series solution obtained using the Laplace Adomian decomposition method. Our findings will inform the development of effective control measures and assist policymakers, healthcare providers, and public health authorities in mitigating the spread and impact of HIV-COVID-19 co-infection.

**Keywords:** COVID-19; HIV; Co-infection; Laplace Adomian decomposition method; series solution; convergence.

# Traveling wave solutions on synergistic co-infection model in crop diseases

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## Abstract

Plants and crops are repeatedly defied by pathogens during their lifespan, causing diseases with a negative impact on the host fitness. In particular, pathogens have caused drastic yield reductions in several important crops. For instance, FAOSTAT database indicates that annually, an yield losse of 20%-40% is due to pests. The majority of plants and crops can be infected by two or more unrelated viruses, leading to more severe symptoms, such as in the case of Maize Lethal Necrosis Disease (MLND) as a synergistic co-infection of Maize Chlorotic Mottle Virus (MCMV) and a potyvirus. See for instance [1, 2]. In this talk, we develop a general spatial explicit synergistic coinfection mathematical model for vector and plant of two virus. The model takes into account the infection through virus only. We then discuss theoretically and numerically the existence of traveling waves solutions. Finally, we discuss possible invasion control through numerical simulation.

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# Global analysis of an SEI plant disease model with saturation incidence rate

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## Abstract

This paper studies an SEI epidemic model for plant disease with saturation incidence rate. Positivity and boundedness of solutions are examined to justify the model biologically. Local stability condition of boundary as well as endemic (positive) equilibrium point is derived. Persistence criterion is developed. The existence of a Hopf bifurcation is shown by considering the intrinsic growth rate as well as disease transmission rate as a bifurcation parameter. Applying Bendixsons criterion for high-dimensional ordinary differential equations, we prove that the endemic equilibrium point is globally stable under certain conditions. The dynamical behaviour of the model is highly sensitive to fluctuations in the disease transmission rate. Finally, numerical simulations are carried out to validate the results obtained.

# A novel investigation of the tuberculosis disease via fractional differential operator

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## Abstract

In this study, a computational solution to the fractional tuberculosis disease is obtained with the help of the Generalized Eulers Method [1, 2]. This article considers two treatment strategies: protective therapy for latent populations and main treatment for infected populations. The compartmental structure of the six-dimensional model includes the susceptible, latent, infected, recovered, and treatment classes. Additionally, we studied the stability of the equilibrium point. To examine the nature of multidimensional differential equations of fractional order that arise in biological illness, the findings produced utilizing the method under consideration are more accurate and straightforward. Additionally, we studied the stability of the equilibrium point. The fractional model has been graphically simulated using MATLAB22.

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# Mathematical Analysis and Parameter Estimation of a Two-Patch Zika Model

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## Abstract

In this work, we developed a multi-patch model for the spread of Zika virus infection taking into account direct and indirect transmissions along with vertical transmission. The model was analyzed to gain some insights into the spread of the disease. The model was fitted to a data set collected from two neighboring countries Brazil and Colombia in order to estimate some of its parameters and use it for calculating the reproduction number and for the purpose of sensitivity analysis. Our results show that the reproduction number is less than one in both countries, which indicates that the disease will die out. Also, our results show that direct transmission is the most important route for the spread of the disease, and hence it has to gain more focus in any controlling strategy.

## Impact of fear of a fractional-order plant disease model with herbivore attack

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### Abstract

This work deals with a fractional-order mathematical model for plants subject to a disease and harvested by herbivores. The existence, uniqueness, non-negativity and boundedness of the considered model has been analyzed. A detailed analysis of different equilibria and their stability has been discussed. Global stability of the interior equilibrium point is investigated. It has been shown that fear factor and fractional order have an important role in the dynamics of system stability. Finally, some numerical simulations are performed to validate our findings.

# Optimal heat transport induced by magnetic nanoparticle delivery in vascularised tumours

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## Abstract

We describe a novel mathematical model for blood flow, delivery of nanoparticles, and heat transport in vascularised tumour tissue. The model, which is derived via the asymptotic homogenisation technique, provides a link between the macroscale behavior of the system and its underlying, tortuous micro-structure, as parametrised in Penta and Ambrosi (2015) [1]. It consists of a double Darcys law, coupled with a double advectiondiffusionreaction system describing heat transport, and an advectiondiffusionreaction equation for transport and adhesion of particles. Particles are assumed sufficiently large and do not extravasate to the tumour interstitial space but blood and heat can be exchanged between the two compartments. Numerical simulations of the model are performed using a finite element method to investigate cancer hyperthermia induced by the application of magnetic field applied to injected iron oxide nanoparticles. Since tumour microvasculature is more tortuous than that of healthy tissue and thus suboptimal in terms of fluid and drug transport, we study the influence of the vessels geometry on tumour temperature. Effective and safe hyperthermia treatment requires tumour temperature within certain target range, generally estimated between  $42^{\circ}C$  and  $46^{\circ}C$ , for a certain target duration, typically 0.5h to 2h. As temperature is difficult to measure in situ, we use our model to determine the ranges of tortuosity of the microvessels, magnetic intensity, injection time, wall shear stress rate, and concentration of nanoparticles required to achieve given target conditions.

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# Traveling wave speed and profile of a go or grow glioblastoma multiforme model

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## Abstract

Glioblastoma multiforme (GBM) is a fast-growing and deadly brain tumor due to its ability to aggressively invade the nearby brain tissue. A host of mathematical models in the form of reaction-diffusion equations have been formulated and studied in order to assist clinical assessment of GBM growth and its treatment prediction. To better understand the speed of GBM growth and form, we propose a two-population reaction-diffusion GBM model based on the go or grow hypothesis. Our model is validated by in vitro data and assumes that tumor cells are more likely to leave and search for better locations when resources are more limited at their current positions. Our findings indicate that the tumor progresses slower than the simpler Fisher model, which is known to overestimate GBM progression. Moreover, we obtain accurate estimations of the traveling wave solution profiles under several plausible GBM cell switching scenarios by applying the approximation method introduced by Canosa.

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# In-depth epidemiology and modeling of the future trends of communicable and non-communicable diseases in Kazakhstan using aggregated big data from the Unified National Electronic Healthcare System

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## Abstract

There is an emerging need to develop a comprehensive epidemiological and indepth analysis of communicable and non-communicable diseases in Kazakhstan. Monitoring electronic health systems (UNEHS) and digitalization of healthcare, disease registries, including most of the emerging diseases that Kazakhstan has implemented. This study has had ultimately aims for the development of a research center for disease control and prevention, using modern epidemiological analytic approaches, advanced biostatistical evaluations, mathematical modeling and machine learning to support with appropriate epidemiologic and methodologic techniques to working on large-scale administrative healthcare data for public health surveillance of infectious diseases. features, particularly, geographical, seasonal and other patterns of communicable and non-communicable diseases using UNEHS. Along with statistical tools, we used mathematical modeling and machine learning algorithms which have provided a better and more comprehensive understanding of the infection transmission dynamics and forecasting the trend of the diseases. Several peer reviewed papers have been published as well as still ongoing many research projects using data from UNEHS and different methodological approaches such as advanced epidemiology, mathematical modeling, and machine learning.

**Funding:** This study was supported by grants from the Nazarbayev University Faculty Development Aggregation and utilization of the large-scale administrative health data in Kazakhstan for population health research and surveillance), Nazarbayev University Faculty Development Research Grant Program modeling of the 10-year trends of cardiovascular diseases and their complications in Kazakhstan using aggregated big data from the united National Electronic Healthcare System) and grant from Ministry

**Keywords:** Big Data in Healthcare, Epidemiology, Machine Learning, Mathematical models

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# Multi-stability analysis of fractional-order quaternion-valued neural networks with time delay

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## Abstract

This paper addresses the problem of multi-stability analysis for fractional-order quaternion-valued neural networks (QVNNs) with time delay. Based on the geometrical properties of activation functions and intermediate value theorem, some conditions are derived for the existence of at least  $(2\mathcal{K}_p^R + 1)^n$ ,  $(2\mathcal{K}_p^I + 1)^n$ ,  $(2\mathcal{K}_p^J + 1)^n$ ,  $(2\mathcal{K}_p^K + 1)^n$  equilibrium points, in which  $[(\mathcal{K}_p^R + 1)^n, (\mathcal{K}_p^I + 1)^n, (\mathcal{K}_p^J + 1)^n, (\mathcal{K}_p^K + 1)^n]$  of them are uniformly stable while the other equilibrium points become unstable. Thus the developed results show that the QVNNs can have more generalized properties than the real-valued neural networks (RVNNs) or complex-valued neural networks (CVNNs). Finally, two simulation results are given to illustrate the effectiveness and validity of our obtained theoretical results.

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# A New Investigation on the Impact of Vaccination on the Transmission of Monkeypox

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## Abstract

The primary aim of this study is to construct a fuzzy fractional mathematical model to investigate the dynamics of monkeypox viral transmission. This dynamical model of monkeypox vaccinations humans is developed using Caputos fuzzy fractional differential equation. Our technique applies to the fuzzy system of fractional ordinary differential equations, and our findings are correct. The impact of the fractional operator on the different compartments of the model has been observed through numerical simulation. The findings demonstrate that maintaining a vaccinated human population mitigates disease transmission. We conclude with short comments and numerical simulations to demonstrate the results.

**Keywords:** monkeypox virus, fractional derivative, fuzzy fractional differential equation, Caputos differential equation.



# Finite-time/fixed-time synchronization of memristive shunting inhibitory cellular neural networks via sliding mode control

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## Abstract

This abstract proposes a novel time-dependent gain parameter-based sliding mode controller (SMC) to realize the finite-time/fixed-time synchronization of memristive shunting inhibitory memristive neural networks (Mem-SICNNs) having time-varying delays. In this regard, a new terminal sliding mode surface is designed and its reachability is analysed. According to synchronization error analysis, the stability property of the desired error system is reached within finite-time/fixed-time range by proposing a unique time-dependent gain parameter-based SMC and choosing the appropriate Lyapunov functionals. Finally, a numerical example is approached by software simulation and manual calculation to estimate the settling-time of the finite-time/fixed-time synchronization criteria of the proposed Mem-SICNNs model.

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**Keywords:** finite-time/fixed-time synchronization; memristor; shunting inhibitory cellular neural networks; sliding mode control.

## Structured dynamics of the cell-cycle at multiple scales

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### Abstract

The eukaryotic cell cycle consists of four distinct phases ( $G_1$ ,  $S$ ,  $G_2$ , and  $M$ ), and it ensures the protection of the cell from DNA-damage and becoming a cancerous cell. DNA-damage activates the regulatory protein p53, which arrests the cell cycle to allow for the repair of this damage, while, p53 is a single inactivated and frequently mutated protein in cancer. As such, this procedure takes place at the cellular level; however, its impact is observed at the level of the entire organism's cell population. In this context, we introduce two models of cell cycle progression: The first model focuses on the cell-scale process of cell cycle progression and the temporal biochemical processes, driven by cyclins, which govern the transition from one phase to another. On the other hand, the second model addresses the broader aspect of cell-cycle progression across a population of cells and its arrest caused by DNA-damage and activation of the p53 gene.

# Asymptotic analysis of Alzheimers disease in thin heterogeneous domains

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## Abstract

We carry out in thin layer the homogenization of a set of Smoluchowskis discrete diffusion-coagulation equations describing the evolution density of diffusion particles that are prone to coagulate in pairs. Alzheimers disease (AD), the most common cause of dementia, destroys cognitive skills and memory as a progressive neurodegenerative disease. The aggregation and the diffusion of  $\beta$ -amyloide peptide ( $A\beta$ ) are responsible to the development of Alzheimers disease at different scales and the thin heterogeneous domains give rise to these phenomena in lower space dimensions. More precisely, assuming that the thin heterogeneous layer is made of microstructures that are distributed inside in a deterministic way including as special cases the periodic and the almost periodic distributions, we make use of the concept of algebras with mean value to state the main compactness results and we also make use of a new approach of the sigma-convergence method that is suitable for the study of such phenomena occurring in thin heterogeneous media. This is made through a systematic study of the sigma-convergence method for thin heterogeneous domains.

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# Control Analysis of *Nilaparvata Lugens* with *Wolbachia* using Sterile Insect Techniques

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## Abstract

Modeling and controlling *Nilaparvata Lugens* (*Lugens*) populations in paddy fields have been highly beneficial to the environment and farmers alike. As part of this study, we describe a sex-structured wild *Lugens* model and male *Lugens* infected with w-Stri (*Wolbachia*). A laboratory test has shown that w-Stri type *Wolbachia* is able to control wild *Lugens* biologically. When male *Lugens* infected with w-Stri mate with wild female *Lugens*, they produce higher levels of cytoplasmic incompatibility (CI). We construct an optimal control problem by using the continuous releasing rate of male *Lugens* infected with w-Stri as a time-dependent control parameter. We examine the optimality in terms of its necessary and sufficient conditions. Furthermore, we discuss the sufficient conditions to reach the elimination of wild *Lugens* through periodic impulsive releases of male *Lugens* infected with wStri. Numerical simulations are performed to validate the theoretical conclusions.

# Cinematographic Method for Mathematical Modeling of Heat Transfer and Exchange Processes in Geometrically Complicated Bodies

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## Abstract

Heat distribution, transfer and exchange processes are widely spread as in nature so in technics. If a region where such a process is to be considered has simple geometric features as prisms, cylinders, balls, classic models mathematical models are effective. In cases when the process occurs in complicated bodies it is preferable an approach of discretization simulating cinematographic motion. This approach allows to calculate heat distribution in metallic grid of a rotating air preheater of energy plants and heat exchange between tube and gas flowing in it when the tube has arbitrary complicated configuration. The main point of the method is a reduction of the problem to linear systems of discrete equations while the most difficulty arises in solving the inverse problem in order to identify the model. It will be discussed questions of developing the method taking into account delay in time and control parameters and game problems.

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## A continuum space is the infinitely great

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### Abstract

An infinitely small quantity is defined as a one-dimensional quantity of finite length but with sizes of space, while an infinitely great quantity is reached by the superposition or accumulation of infinitely many finite quantities, by the way of the change in direction. The change in direction indicates that there is a jump from a finite quantity to infinitely many finite quantities (infinitely great). The form of the manifestation of the infinitely great is one quantitative continuum that cannot be operated by any algorithms and all parts of space we see is this one quantitative continuum. Any value are this single quantitative continuum that indicates the infinitely great and compresses any quantities outside of it to nothing. As a result, the infinite exact value of a circumferential length ( $\pi$ ) has been obtained here.

**Keywords:** infinitely great, change in direction, one quantitative continuum

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## Degree Reduction of Bezier Curves

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### Abstract

There are substantial methods of degree reduction in the literature. Existing methods share some common limitations, such as lack of geometric continuity, complex computations, and one-degree reduction at a time. In this talk, an approximate geometric multi-degree reduction algorithm of Bezier curves is proposed. Geometric continuity conditions are applied in the degree reduction process to preserve the boundary control points. The general equation for high-order (G2 and above) multi-degree reduction algorithms is nonlinear, and the solutions of these nonlinear systems are quite expensive. In this paper,  $G_1$ -continuity conditions are imposed besides the  $G_0$ -continuity conditions. While some existing methods only achieve the multi-degree reduction by repeating the one-degree reduction method recursively, the proposed method achieves multi-degree reduction at once. The distance between the original curve and the degree-reduced curve is measured with the 1-infinite norm. Numerical examples and figures are presented to state the adequacy of the algorithm. The proposed method not only outperforms the existing methods of degree reduction but also guarantee geometric continuity conditions at the boundary points, which is very important in CAD and geometric modeling.

## Role of the NFAT-signaling network in controlling allergic skin reactions

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### Abstract

Allergic contact dermatitis (ACD) is a type of human skin allergic reactions caused by multiple contact allergens. It accounts for at least 20% which stresses on the need of basic and clinical research on it. So far, a plethora of data supports a major role of CD4+ and CD8+ T lymphocytes in the progression and maintenance of disease. However, in-depth understanding about the molecular mechanisms of its pathophysiology is still limited. The Ca<sup>++</sup>-Calcineurin-NFAT-signalling network is an essential mediator of T cell receptor signaling. Out of four NFATc members, NFATc1 plays a prominent role in T cell activation. Therefore, we investigated its role in contact hypersensitivity (CHS) which is an established animal model to study human ACD. The 2,4,6- trinitrochlorobenzene (TNCB)-induced CHS response was diminished in *Nfatc1<sup>fl/fl</sup>Cd4-cre* mice (*Nfatc1<sup>-/-</sup>*) as compared to wild-type (WT) mice. The reduced CHS response was linked to a lower percentage of interleukin (IL)17-producing CD8+ T (Tc17) cells in inflamed skin and draining lymph nodes (dLN). In vitro Tc17 polarization assays displayed that *Nfatc1<sup>-/-</sup>* CD8+ T cells have a reduced ability to polarize into Tc17 cells. In the absence of NFATc1, CD8+ T cells favor the differentiation of Interferon (IFN)- $\gamma$  secreting CD8+ T (Tc1) cells, whereas in its presence they develop into Tc17 cells. Moreover, the CHS response was rescued in nave *Nfatc1<sup>-/-</sup>* mice by adoptive transfer of TNCB-sensitized WT CD8+ T cells. Finally, we detected the infiltration of Tc17 cells in human ACD skin samples by immunofluorescence analysis, supporting a link to the CHS model. By controlling Tc17 generation, our data showed that NFATc1 acts a molecular switch in the development of Tc17 cells. Consequently, it controls the CD8+ T cell-mediated allergic skin inflammation. These findings will pave the way for future innovative therapeutic options for the treatment of allergic immune responses.



# Finite-time and fixed-time synchronization control of fuzzy Shunting Inhibitory Cellular Neural Network

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## Abstract

Several problems based on finite-time synchronization are explored in this paper. The primary objective is to design the controllers for the various models based on Fuzzy Shunting Inhibitory Cellular Neural Networks (FSICNNs) and find out sufficient conditions for the systems' solutions to reach synchronization in finite time. In other words, we prove the existence of finite-time synchronization for three FSICNNs models that have not been studied before and suggest both the controllers and Lyapunov functions that would yield the feasible convergence time between solutions. Namely, we study models of delayed FSICNNs with and without inertial terms and FSICNNs with diffusion and without delays. Derived by means of the well-known in the scope of calculus maximum-value approach criteria, we give an upper bound to the time by which the synchronization phenomenon is guaranteed to occur in the three FSICNNs models. These results are supported by computer simulations showing the solutions' behavior for different initial conditions of FSICNNs.

# On the Hyers-Ulam stability and the Hyers-Ulam- Rassias stability of nonlinear iterative integro- differential equations with variable delays

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## Abstract

This work is interested in the existence and uniqueness of solutions, the Hyers-Ulam stability (HUS) and the Hyers-Ulam- Rassias stability (HURS) of an iterative integrofunctional differential equation (IIDDE) of first order with variable delays on a bounded interval, half infinite intervals and an infinite interval using the Banach fixed point theorem. The outcomes of this paper have scientific novelties and provide new contributions to the topic of the existence and uniqueness of solutions and the Ulam stabilities of IIDDEs.

**Keywords:** Uniqueness of solutions, Ulam stabilities, IIDDEs, fixed point theorem

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# New Results on Integro-Differential Equations with Variable Retardation

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## Abstract

The paper is interested in a class of nonlinear time varying retarded integro- differential equations (RIDEs). By the functional Lyapunov-Krasovskii method, two new results having weaker conditions related to the uniformly stability (US), uniformly asymptotic stability (UAS), integrability, boundedness and boundedness at infinity of solutions of that RIDEs are given. We define a Lyapunov Krasovskii functional (LKF) for that system of RIDEs and perform the proof of the given results. Two examples are provided with their graphs to affirm the proposed theorems. The study of the results of this paper shows that the proposed theorems are not only applicable to the time varying linear RIDEs, but also applicable to the time varying nonlinear RIDEs.

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# Investigation to Explore the Relationship Between Awareness and Behavior Change Towards NPI and Vaccine Usage in the Transmission of COVID-19 in Thailand

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## Abstract

The novel coronavirus has caused a worldwide pandemic and emphasizes the need for mathematical modeling to understand the transmission dynamics of the virus and guide effective public health strategies. In this paper, we develop a compartmental model to study the transmission dynamics of COVID-19 in Thailand. Our model considers the vaccine population, divides the infected population into two groups symptomatic and asymptomatic, the impact of non-pharmaceutical interventions (NPIs), and awareness on reducing the infectiousness of the population. The model is fitted using real data from Thailand and we perform qualitative analysis to examine the stability of the equilibrium point and determine the conditions that lead to asymptomatic stability of the corresponding endemic equilibrium point. We also convert the model to the Atangana-Baleanu fractional order and carry out numerical simulations. We conduct sensitivity analysis on the models basic reproduction number to identify the parameters that have the greatest impact on reducing its value, including contact, recovery, contagiousness, and education outreach or campaign. Our analysis reveals that a combination of strategies, including increased education outreach and NPI usage, can effectively curb the spread of COVID-19 in Thailand. Our findings highlight the importance of adapting ongoing public health strategies to combat the spread of COVID-19. Our conclusion is that mathematical modeling can play a critical role in guiding effective public health interventions to mitigate the impact of pandemics such as COVID-19.

**Keywords:** COVID-19, Equilibrium points, Stability analysis, Atangana-Baleanu, Sensitivity analysis

# Modeling the effect of radio-chemotherapy on tumor cells interaction with optimal control and global sensitivity analysis

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## Abstract

Oncologists and applied mathematicians are interested in understanding the dynamics of cancer-immune interactions, mainly due to the unpredictable nature of tumour cell proliferation. In this regard, mathematical modelling offers a promising approach to comprehend this potentially harmful aspect of biology. This paper presents a novel dynamical model that incorporates the interactions between cancer cells, healthy tissue cells, and immune-stimulated cells in the environment of radio-chemotherapy treatment. We analyze the equilibria, identify conditions that lead to chaotic dynamics, and rigorously demonstrate the existence of chaos. Local stability analysis is performed, and we investigate transcritical, saddle-node, and Hopf bifurcations analytically and numerically. We derive the stability and direction conditions for periodic solutions. Furthermore, we examine an optimal control problem in the model that describes the dynamics of tumour-immune interactions, considering treatments such as radiotherapy and chemotherapy as control parameters. Our goal is to utilize optimal control theory to reduce the cost of radio chemotherapy, minimize the harmful effects of medications on the body, and mitigate the burden of cancer cells by maintaining a sufficient population of healthy cells. Cost-effectiveness analysis is employed to identify the most economical strategy for reducing the disease burden. Additionally, we conduct a Latin hypercube sampling-based uncertainty analysis to observe the impact of parameter uncertainties on tumour growth, followed by a sensitivity analysis. Through computer simulations, we demonstrate how the dynamical behaviour changes with variations in system parameters. The numerical results validate the analytical findings and illustrate that a multi-therapeutic treatment plan can effectively reduce tumour burden within a few days of drug administration

## Inverse Problems in Delay Differential Equation

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### Abstract

The aim of the research is to introduce concepts of direct problems, moment problems and inverse problems in delay differential equation in oscillating material i.e vibrating string and electric circuit by using finite difference scheme derived from Taylor series. We are to find the unknown properties of vibrating string which are usually given while solving direct problems.

**Keywords:** Inverse problems, direct problems, moment problems, delay differential equation, Oscillating system, electric circuit.

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# Hybrid Crossover Dynamics of Immuno-Chemotherapy with Gene Therapy and Time-Delay Model: Numerical Treatments

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## Abstract

Piecewise fractional differential equations (deterministic-stochastic differential equations or vice versa) have been introduced recently in literature. The purpose of this piecewise approach is to study effectively the model with real data. In this paper, a mathematical model of immuno-chemotherapy with gene therapy and the time delay is extended using the stochastic-deterministic piecewise hybrid fractional derivatives, where the hybrid fractional order and variable-order fractional operators are applied to extend the deterministic models. Two numerical methods are constructed to study the behavior of the proposed models. These methods are the nonstandard Caputo proportional constant finite difference method for solving the hybrid fractional deterministic models and the nonstandard Euler Maruyama technique to solve the stochastic model. Several numerical tests demonstrate the efficiency of the method and support the theoretical results.

# Investigation of The stability of The $F^*$ Iterative Algorithm on Strong Pseudocontractive Mappings And Its Applications

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## Abstract

This paper is centered on conducting an inquiry into the stability of the  $F^*$  iterative algorithm to the fixed point of a strongly pseudo-contractive mapping in the framework of uniformly convex Banach spaces. To achieve the desired result, certain existing inequalities in convex Banach spaces were utilized as well as the stability criteria of Harder and Hicks. Other necessary conditions for the stability of the  $F^*$  algorithm on strong pseudo-contractive mapping were also obtained. Through a numerical approach, we prove that the  $F^*$  iterative algorithm is H-stable for strongly pseudo-contractive mapping. Finally, the solution of the mixed-type Volterra-Fredholm functional non linear integral equation is estimated using our results.

**Keywords and Phrases :** Stability,  $F^*$ -iterative algorithm, Pseudo-contractive mappings, nniformly convex Banach space, Mixed-type Volterra-Fredholm integral equation.



# Impulsive Delay Differential Equations with Delayed-Effective Inputs

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## Abstract

It is well known that time delays exist in biological systems. Interventions are essential in many cases, such as drug administrations, fishery managements, vaccinations, etc. Current treatments for such interventions in mathematical modeling simulations and control theory are through impulse of perturbations of specific state variables and taking immediate effects, even though the effects of such interventions are delayed in most cases. To study the delayed effect of the biological interventions, we establish a novel concept for delay differential equation system with functional impulsive inputs that take effects in a future finite time interval. In this talk, we showcase this new modeling framework with the Hutchinson-Wright logistic population model, and the insulin administrations in artificial pancreas systems with a single pump (for insulin administration) or dual pumps (for both insulin and glucagon administrations).

# Bifurcation in a delayed predatorprey model with Holling type IV functional response incorporating hunting cooperation and fear effect

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## Abstract

In this paper, we propose and analyze a delayed predatorprey model with Holling type IV functional response taking into account the cooperative hunting behavior in predators and the fear effect on prey due to predation risk. We first investigate the existence and stability of the model equilibria and show that the non-delayed model may lose its stability behavior via Hopf bifurcation by considering hunting cooperation as a bifurcation parameter. Next, we focus on the impact of fear on the dynamics of the delayed model and find that a Hopf bifurcation occurs for the delay as a bifurcation parameter. Furthermore, we numerically prove that the time-delay parameter may switch the stability behavior of the system. Before closing our study, numerical simulations are performed to illustrate our analytical findings.

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# Sensitivity Analysis and Optimal Control of Serial Killing

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## Abstract

This work focuses on the elimination of Serial Killing by utilizing minimum resources. The initial crime transmission rate is examined using a mathematical model. We perform sensitivity analysis of the reproduction number  $\mathcal{R}$ , to find the most active parameters regarding the transmission of crime. These active parameters are intervened using suitable control variables. The objective function is designed to control the crime-addicted classes at a minimum cost. To minimize the objective functional Lagrangian and Hamiltonian are used. To check the validity of the results numerical simulations through MATLAB are performed.

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# Analysis and stability of a social interaction model with age-structure and law enforcement

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## Abstract

In this work, we consider a social interaction model with age-structure and law enforcement. It is evident that age is an important factor in social dynamical models. We study stability of equilibrium solutions. This analysis is performed using basic reproduction number. Moreover, we study optimal control of considered problem. At the end, graphs are provided for illustration.

# Some Recent Studies Using Fractional Operators on the Incomplete Special Functions

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## Abstract

Fractional calculus has numerous and varied applications in the domains of engineering and science, including optics, signals processing, viscoelasticity, fluid mechanics, electrochemistry, biological population models, and electromagnetics [2]. At this moment, the fractional calculus has opened its wings even larger to cover the dynamics of complex real world and new ideas are starting to be implemented and tested on real data. In order to better understand physical and engineering processes, fractional differential equations have been utilized to simulate them. In this study, the generalised fractional calculus operators will be examined and further formulate on the incomplete special functions [1]. The findings are presented in a concise manner that are helpful in creating certain lists of fractional calculus operators. A significant number of new and recognised outcomes including particularly the incomplete H-function and incomplete I-function follow as special instances of the primary discoveries.

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# A Seasonally Forced Eco-Epidemic Predator-Prey Model for the Impacts of Fear and Its Carry-Over Effect with Selective Predation

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## Abstract

In many ecological situations, predation fear and its carry-over effect play crucial role in a predator-prey system. Predators have ability to distinguish prey that have been infected by some parasite, and have tendency to avoid the infected prey to reduce fitness cost. Here, we propose an eco-epidemic predator-prey model incorporating selective predation induced fear and its carryover effect. Also assume that the disease is of SI-type. In analytical part, we discuss basic properties like boundedness, feasible equilibria and its stability, and existence of Hopf bifurcation by taking carry-over effect as possible bifurcation parameter. Analytical findings are validated through numerical simulation. Our simulation results show that both the fear and its carry over effect destabilizes the system through limit cycle oscillations. Whereas, another fear factors: fear due to disease transmission and death of infected prey stabilizes the system by terminating the oscillations. To model the above situations more realistically, we consider the non-autonomous system by taking the parameters representing the cost of fear, carry-over effect, disease prevalence and selective preference of predators vary with time. For the seasonally forced system, conditions are obtained for which the system has at least one positive periodic solution. Our seasonally forced model showcases the appearance of a unique periodic solution, higher periodic solutions and complex bursting patterns

# New properties of the oscillatory behavior of neutral differential equations of higher order

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## Abstract

In this research, we applied three techniques: comparison technique, Riccati technique and integral averages technique to obtain several conditions and properties of the oscillatory behavior of neutral differential equations of higher-order. As proof of the effectiveness of new conditions and properties, we present some examples.

**Keywords:** Oscillation criteria, Higher order, Differential equations

# Optimal Control of an Influenza Transmission Model with Vaccination and Therapy

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## Abstract

The present work included the formulation and analysis of a non-linear epidemic mathematical model that incorporates the influence of vaccination and therapy in the context of Influenza transmission. This paper examines the stability of the disease-free equilibrium at both the local and global levels. Additionally, the model's fundamental reproduction number,  $\mathcal{R}_0$ , is calculated. The use of the normalised forward sensitivity index approach is being employed to ascertain the sensitivity analysis of the parameters in relation to the variable  $R_0$ . This study investigates the influence of vaccination and treatment rates on populations affected by infections. Two distinct types of controls are used to enhance the applicability of this model to an optimum control situation. The first control strategy aims to maintain the vaccination of persons who remain vulnerable, while the second control strategy seeks to enhance the treatment options available for affected individuals. The use of Pontryagin's Maximum Principle is employed for the purpose of analysing and numerically resolving the optimum control issue. Subsequently, the efficacy of the optimal control is evaluated by comparing the levels of the infected population with and without the implementation of controls. The integration of therapy and immunisation tactics has shown more favourable results, as it effectively reduces the prevalence of the illness within the designated period of control. **Keywords:** mathematical model, stability analysis, sensitive analysis, optimal control



# Multiple Periodic Solutions for Delay Differential Equations

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## Abstract

In this work, we discuss the existence and the multiplicity of nonconstant subharmonic solutions for the following problem,

$$\begin{aligned}x''(t) + \lambda f(t, x(t), x(t - \tau)) &= 0, & t \in \mathbb{R}/\{t_j\}, \\-\Delta x'(t_j) &= I_j(x(t_j)), & j \in \mathbb{Z}, \\x(t) - x(t + 2r) &= x'(t) - x'(t + 2r) = 0,\end{aligned}\tag{5.1}$$

where  $r > 0$  and  $\lambda > 0$  are given constants,  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  is  $r$ -periodic in  $t$ .  $\Delta x'(t_j) = x'(t_j^+) - x'(t_j^-)$  with  $x'(t_j^\pm) = \lim_{t \rightarrow t_j^\pm} x'(t)$ ;  $t_j, j \in \mathbb{Z}$ , the instants where the impulses occur are such that  $0 = t_0 < t_1 < t_2 < \dots < t_p < t_{p+1} = 2r, t_{j+p+1} = t_j + 2r$ , for some fixed  $p \in \mathbb{N}$  and the function  $I_j : \mathbb{R} \rightarrow \mathbb{R}$  is continuous and satisfies  $I_{j+p+1} = I_j$ , for all  $j \in \mathbb{Z}$ . To study the solvability of (5.1), we use a variational approach based on an application of a variant of mountain-pass theorem.

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## Optimal Control of a Periodically Switched Epidemic Model

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### Abstract

In this paper, we examine a modified Susceptible-Infected-Treatment-Recovered (SITR) model. The SITR model is assumed to be a periodically switched system. When  $\mathcal{R}_0$  is greater than 1, the control measures such as vaccination, isolation, and specific treatments are implemented. Given the limited availability of these resources, it is crucial to use them in the most efficient manner. The aim is to study the optimization of control strategies for infectious disease outbreaks in dynamic and resource-limited environments. In the first part of the article, we analyze the stability of the switched model and then formulate an optimal control problem of the periodically switched system. In the end, numerical simulations are performed to support the results discussed.

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# Stability and Bifurcation Analysis of an SIR Epidemic Model

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## Abstract

Throughout history, people have faced various epidemics, and the societal impacts caused by these epidemics have often been understood through mathematical modeling. Understanding and controlling the dynamics of disease spread represent fundamental challenges in the field of medical sciences. Mathematical epidemiology constitutes a significant component of research in this field. Especially in recent years, the outbreak of global pandemics like COVID-19 has further emphasized the importance of mathematical epidemiology. Research in this field plays a critical role in developing strategies to control and prevent epidemics. In this talk, we will consider a class of SIR epidemic model that divides the population into three classes, namely, susceptible, infected, and recovered individuals. First, we will demonstrate that the model is well-defined biologically. Thus, we will study dynamics analysis of the proposed model. We also anticipate that this model can be used to investigate the dynamics of various diseases. Finally, we will support our original and theoretical results with numerical examples.

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# On the Nonstandard Numerical Discretization of an SIR Epidemic Model

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## Abstract

Epidemiological models are often formulated as systems of non-linear differential or difference equations. When a model represented by a nonlinear system of differential equations is given, this model can be discretized using certain methods. It is important to ensure that the discretized model retains as many dynamical properties of the continuous model as possible. For the purpose of discretization, Euler and Runge-Kutta methods, along with numerous other finitedifference techniques, are commonly used. In this talk, we will propose a class of discrete SIR model with vaccination. A nonstandard finite difference scheme (NSFD) will be developed. We will investigate the dynamical properties of discrete SIR epidemic model, including the nonnegativity and boundedness of solutions, the existence of equilibria, and the computation of the basic reproduction number. The local and global asymptotic stability of disease-free and endemic equilibria will be studied. Finally, some numerical simulations have been provided to illustrate our theoretical results.

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# Numerical method for solving integro-differential model of biological species living together

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## Abstract

In this article, we investigate the solution of biological species living together model using the operational matrix method. This model consists of system of two fractional nonlinear integro-differential equations. We modify the operational matrix method to fit with this type of systems. We simplify the resulting algebraic system in practicable way. Instead of solving system of  $2m \times 2m$  equations, we simplify the problem to find the solution of  $2 \times 2$  of nonlinear equations. This for sure will reduce the computational cost when  $m$  is large. We proved the existence and uniqueness of the solution of biological species living together model. Also, we investigate two types of stability which are Ulam-Hyers stable and the generalized Ulam-Hyers stable. We implement the proposed method to solve three problems. We compare our results with the existence results in the literature when  $\alpha = 1$ . Also, we study the convergence of the solution when  $\alpha$  approaches to 1. The results show the efficiency of the proposed method.

## Problem with integral conditions for system of partial differential equations of third order

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### Abstract

Let  $H(\mathbb{R}_+ \times \mathbb{R}^n)$  be a class of entire functions on  $\mathbb{R}$ ,  $K_L$  is a class of quasipolynomials of the form  $\varphi(x) = \sum_{i=1}^n Q_i(x) \exp[\alpha_i x]$ , where  $\alpha_i \in L \subseteq \mathbb{C}$ ,  $\alpha_k \neq \alpha_l$ , for  $k \neq l$ ,  $Q_i(x)$  are given polynomials.

Each quasipolynomial defines a differential operator  $\varphi\left(\frac{\partial}{\partial \lambda}\right)$  of finite order on the class of entire function, in the form

$$\sum_{i=1}^m Q_i\left(\frac{\partial}{\partial \lambda}\right) \exp\left[\alpha_i \frac{\partial}{\partial \lambda}\right] \Big|_{\lambda=0}.$$

In the strip  $\Omega = \{(t, x) \in \mathbb{R}^{n+1} : t \in ([T_1, T_2] \cup [T_3, T_4]), x \in \mathbb{R}^n\}$  we consider of the system of equations

$$\frac{\partial^3 U_i}{\partial t^3} + \sum_{j=1}^n \left\{ a_{ij} \left(\frac{\partial}{\partial x}\right) \frac{\partial^2}{\partial t^2} + b_{ij} \left(\frac{\partial}{\partial x}\right) \frac{\partial}{\partial t} + c_{ij} \left(\frac{\partial}{\partial x}\right) \right\} U_j(t, x) = 0, \quad (5.2)$$

$$\int_{T_1}^{T_2} e^{\alpha_k t} U_{ik}(t, x) dt + \int_{T_3}^{T_4} e^{\beta_k t} U_{ik}(t, x) dt = \varphi_{ik}(x), \quad k = 1, 2, 3 \quad (5.3)$$

where  $a_{ij}\left(\frac{\partial}{\partial x}\right)$ ,  $b_{ij}\left(\frac{\partial}{\partial x}\right)$ ,  $c_{ij}\left(\frac{\partial}{\partial x}\right)$ , for  $i = 1, \dots, n$ , are differential expression with entire symbols  $a_{ij}(\lambda) \neq 0$ ,  $b_{ij}(\lambda) \neq 0$ ,  $c_{ij}(\lambda) \neq 0$ ,  $\alpha, \beta \in \mathbb{R}$ . Let be  $\eta(\lambda) = \int_{T_1}^{T_2} W^{n-1}(t, \lambda) dt + \int_{T_3}^{T_4} W^{n-1}(t, \lambda) dt$  is a certain function  $W(t, \lambda)$  is a solution of equation  $\left(\frac{d^n}{dt^n} + \sum_{i=1}^n a_i(\lambda) \frac{d^{n-i}}{dt^{n-i}}\right) W(t, \lambda) = 0$ , satisfies conditions  $W^n(t, \lambda) \Big|_{t=0} = 1$ ,  $W^{n-1}(t, \lambda) \Big|_{t=0} = W(t, \lambda) \Big|_{t=0} = 0$ .

Denote be  $P = \left\{ \eta(\lambda) = 0, \lambda \in \mathbb{C} \right\}$  set zeros of function  $\eta(\lambda)$ .

**Theorem.** Let  $\varphi_{ki}(x) \in K_L$ ,  $i = 1, \dots, n$ ,  $k = 1, 2, 3$ ,  $j = 1, \dots, n$  then the class  $K_{L \setminus P}$  exist and unique solution of the problem (1) - (2). Solution of the problem (1) - (2) can be represented in the form

$$U_i(t, x) = \sum_{k=1}^3 \sum_{p=1}^n \varphi_{kp} \left(\frac{\partial}{\partial x}\right) \left\{ \frac{1}{\eta(\lambda)} T_{kjp}(t, \lambda) \exp[\lambda x] \right\} \Big|_{\lambda=0},$$

where  $T_{kjp}(t, \lambda) = l^T \left(\frac{d}{dt}, \lambda\right)$  is transpose of a matrix  $\left(\frac{d}{dt}, \lambda\right)$ .

Solution of the problem (1) - (2) according to the differential-symbol method [1] and [2] exists and unique in the class of quasi-polynomials.

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## Problem with integral conditions for system of evolution equations of third order

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### Abstract

Let  $B$  be a Banach space,  $A : B \rightarrow B$  linear operator, for this operator arbitrary powers  $A^n$ ,  $n = 2, 3, \dots$  be also defined in  $B$ , denote by  $x(\lambda)$  the eigenvector of the operator  $A$ , which corresponds to its eigenvalue  $\lambda \in \Lambda \subseteq \mathbb{C}$ .

We consider of the system of equations

$$\frac{d^3 U_i}{dt^3} + \sum_{j=1}^n \left\{ a_{j,i}(A) \frac{d^2}{dt^2} + b_{j,i}(A) \frac{d}{dt} + c_{j,i}(A) \right\} U_j(t) = 0, \quad (5.4)$$

$$\int_{T_1}^{T_2} t^k U_{i,k}(t) dt + \int_{T_3}^{T_4} t^k U_{i,k}(t) dt = \varphi_{i,k}, \quad i = 1, \dots, n, k = 1, 2, 3. \quad (5.5)$$

where  $a_{j,i}(A), b_{j,i}(A), c_{j,i}(A)$  are abstract operators, with entire symbols  $a_{j,i}(\lambda), b_{j,i}(\lambda), c_{j,i}(\lambda) \neq \text{const}$ , for  $\lambda \in \Lambda$ ,  $\varphi_{i,k} \in B$ ,  $T_1 > T_2 > T_3, T_4 >$ ,  $([T_1, T_2]) \cup ([T_3, T_4]) \subset \mathbb{R}$ .

Solution of the problem (1), (2), (3) according to the differential-symbol method [1] exists and unique in the class of quasi-polynomials.

Solution of the problem(1) -(2) according to the differential-symbol method [1] and [2] exists and unique in the class of quasi-polynomials.

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# Stability and Hopf bifurcation analysis in a discrete and distributed delayed neural network model with inertial coupling

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## Abstract

This talk presents the investigation of stability and Hopf bifurcation in a neural network model featuring discrete and distributed delays for a pair of neurons with inertial coupling. For a general distribution kernel, we identify a subset of amplitude death region. To delineate the complete region of amplitude death, we explore various delay distributions, including Dirac delta functions and gamma distributions. We establish stability criteria for the trivial steady state within parameter spaces encompassing neural interaction history and coupling strength. Our analysis reveals the occurrence of both Hopf and steadystate bifurcations as the steady state loses stability. Employing normal form theory and center manifold theory, we derive explicit formulas to examine the stability, direction, and other characteristics of periodic solutions emanating from the Hopf bifurcation points. To validate the theoretical findings, we provide a numerical illustration.

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