

# **Delay Differential Equations: Theory, Applications and New Trends (DDEs-TANTs)**

Workshop from 3–4 Oct., 2018, United Arab Emirates University, Al-Ain, UAE

## **Book of Abstracts**

UNITED ARAB EMIRATES UNIVERSITY



College of Science  
Department of Mathematical Sciences  
UAE  
3–4 October, 2018

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## 1 Organizing Committee

1. Prof. Fathalla A. Rihan (Chair, UAEU), Email: frihan@uaeu.ac.ae
2. Dr. Nasser Al-Salti (SQU, OMAN)
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5. Dr. Sehjeong Kim (UAEU)

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3. Ms. Hebatallah J. Alsakaji (PhD student, UAEU)

## 2 Workshop Overview

Recently, there has been increasing interest related to the theory of delay differential equations (DDEs), due to their important applications in physics, biology, ecology, and physiology. The main objective of this well-focused workshop (DDEs-TANTs 18) is to provide an opportunity to study the new trends and analytical insights of the delay differential equations, existence and uniqueness of the solutions, boundedness and persistence, oscillatory behavior of the solutions, stability and

bifurcation analysis, parameter estimations and sensitivity analysis, and numerical investigations of solutions. Potential topics include but are not limited to the following: (i) Development of novel theories or improvement to existing theories on delay differential equations; (ii) Development of novel numerical approaches for delay differential equations; (iii) Stability, sensitivity analysis and bifurcation analysis of time delay models; (iv) Optimal control of nonlinear delay differential systems.

The scientific program of the workshop consists of 50-minute plenary lectures, 30-minute invited lectures and 20-minute contributed talks.

The selected papers presented at DDEs-TANRs workshop will be published in a special issue of the journal "Discrete & Continuous Dynamical Systems - S (DCDS-S)", one of the American Institute of Mathematical Series.

### 3 Dedication

This workshop and special issue are dedicated to the memory of Professor Christopher T. H. Baker (1939–2017).

### 4 Keynote Speakers

1. Prof. Gennady Bocharov, Institute of Numerical Mathematics, Russian Academy of Sciences, Moscow, Russia;
2. Prof. Yang Kuang, School of Math and Statistical Sciences, Arizona State University, USA;
3. Prof. Radouane Yafia, Ibn Zohr University, Morocco.

#### Scientific Committee:

- |                                  |                                   |
|----------------------------------|-----------------------------------|
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| 2. Prof. G. Bocharov (Russia)    | 7. Prof. Radouane Yafia (Morocco) |
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| 4. Prof Hongjiong Tian (China)   | 9. Dr. I. M. Elmojtaba (Oman)     |
| 5. Prof. Haydar Akca (UAEU)      | 10. Prof. Cemil Tunc (Turkey)     |

## 5 Abstracts

# Delay differential equations for modelling and control of virus infection dynamics

G. Bocharov<sup>1</sup>

<sup>1</sup> Marchuk Institute of Numerical Mathematics, Russian Academy of Sciences, Moscow, Russian Federation

Email: bocharov@m.inm.ras.ru

### Abstract

Systems of delay differential equations (DDE) are widely used for studying the dynamics of virus infections and immune responses as individual models or building blocks of hybrid multi-scale models [1]. Virus diseases are characterized by a variety of courses and outcomes. Understanding the mechanisms of chronic infections and elaborating approaches to their treatment is central to mathematical immunology. Effective treatment of infections (e.g., HIV) requires application of multiple drugs acting on the virus- and the host organism physiology. Earlier, for the DDE model of infectious disease Guri I. Marchuk proposed an approach to cure the chronic infections via perturbing the respective steady states. We present a computational approach to designing multi-modal perturbations using the so called, optimal disturbances [2]. The optimal disturbances of the steady states are computed by maximizing the perturbation-induced response. The control approach is illustrated for a multi-stable DDE system modeling the experimental LCMV infection. The optimal disturbances of high viral load steady state driving it to a low viral load steady state are computed. Comparison of the optimal disturbances obtained using either the  $L_2$ - or  $W_2^1$  norm is presented [3]. It is argued that optimal disturbances computed using the Sobolev norm are superior for designing multi-modal therapies.

The research was funded by the Russian Science Foundation (Grant no. 18-11-00171).

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# Oscillatory dynamics of an intravenous glucose tolerance test model with delay interval

Y. Kuang<sup>1</sup>

<sup>1</sup>School of Mathematical and Statistical Sciences, Arizona State University, Tempe, Arizona, USA

Email: kuang@asu.edu

## Abstract

Type 2 diabetes mellitus (T2DM) has become prevalent pandemic disease largely due to the modern life style. The intravenous glucose tolerance test (IVGTT) is an effective protocol to determine the insulin sensitivity, glucose effectiveness and pancreatic  $\beta$ -cell functionality, through the analysis and parameter estimation of a proper differential equation model. In this talk we propose a novel approach to model the time delay in IVGTT modeling. This approach uses two parameters to simulate not only both discrete time delay and distributed time delay in the past interval, but also the time delay distributed in a past sub-interval. Longer time delay, either a discrete or distributed delay, often destabilize a system. This may not be true for time delay over a sub-interval. We present analytically some basic model properties which are desirable biologically and mathematically. We show that this relatively simple model provides good fit to fluctuating patient data sets and reveals some intriguing dynamics. Moreover, our numerical simulation results indicate that our model may remove the defect in well known Minimal Model which often overestimates the glucose effectiveness index.

## References

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# Hopf Bifurcation in Delay Differential Equations and Applications

R. Yafia<sup>1</sup>

<sup>1</sup>Department of Mathematics, Ibn Zohr University, CST Ait Melloul, Agadir, Morocco  
Email: r.yafia@uiz.ac.ma

## Abstract

This talk is concerned with the Hopf-bifurcation phenomenon in delayed differential equations, which occurs naturally in biology, ecology and physics etc. Based on the reduction of the abstract delay differential equation to a two-dimensional system via the variation of constant formula and the center manifold Theorem and estimating the distance between solutions of the original equation and the bifurcating periodic solutions, we estimate the stability region of bifurcating branch. Considering time delay as a parameter of bifurcation in a delayed system of differential equations which modeling the interaction between two species in population dynamics. We analyze the local stability/instability of the possible equilibrium points and the occurrence of Hopf bifurcation for some critical value of time delay. In the end, we prove the stability/instability of the bifurcating periodic solutions via normal form Theory and center manifold Theorem.

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# Qualitative and Quantitative Features of Delay Differential Equations in Biosciences

Fathalla A. Rihan<sup>1</sup>

<sup>1</sup>Department of Mathematical Sciences, College of Science, UAE University, 15551, Al Ain, UAE

Email: frihan@uaeu.ac.ae

## Abstract

Delay differential equations (DDEs) are a class of differential equations that have received considerable attention and been shown to model many real life problems, traditionally formulated by systems of ordinary differential equations (ODEs), more naturally and more accurately. Such class of DDEs are widely used for analysis and predictions of systems with memory such as population dynamics, epidemiology, immunology, physiology, neural networks and other biological and physical systems with memory. The aim of this talk is to provide a wide range of DDEs and show that they have a richer mathematical framework for the analysis of dynamical systems. We present suitable numerical techniques for solving the forward and backward problems of DDEs. Sensitivity analysis is also investigated for DDEs.

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# Continuous Block for Ordinary and Delay Differential Equations

H. Tian

Department of Mathematics, Shanghai Normal University, Shanghai, China

## Abstract

Continuous numerical methods have many applications in the numerical solution of discontinuous ODEs, DDEs, NDDEs, IDEs, etc. We are concerned with continuous extensions for the discrete approximate solutions of ODEs generated by block methods. Existence, uniqueness, convergence and stability of the continuous extensions for ODEs are discussed. As an application, we adopt the continuous methods to solve DDEs and establish their numerical analysis. Several numerical experiments are given to illustrate the performance of the continuous block methods.

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# Delay differential equations: a case study in malaria transmission dynamics

Ephraim Agyingi

School of Mathematical Sciences, Rochester Institute of Technology, Rochester,  
New York, USA

Email: eoasma@rit.edu

## Abstract

We consider coupled systems of delay differential equations arising from a mathematical model of the transmission dynamics of malaria species. We analyze the equilibria of the system for stability and bifurcation. Using the reproduction number of the system as a threshold parameter, we ascertain global asymptotic stability for the disease free equilibrium, stability switches and Hopf bifurcations for the non-disease free equilibrium

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# Delay Differential Model with Optimal Control for a Cancer Treatment Based on Synergy Between Anti-angiogenic and Immune Cell Therapies

N.H. Sweilam<sup>1</sup>, F.A. Rihan<sup>2</sup>, S. M. El-Mekhlafi<sup>3</sup>

<sup>1</sup>Department of Mathematics, Faculty of Science, Cairo University, Egypt

<sup>2</sup>Department of Mathematical Sciences, College of Science, UAE University, 15551, Al Ain, UAE

<sup>3</sup>Department of Mathematics, Faculty of Education, Sana'a University, Yemen

Email: nsweilam@sci.cu.edu.eg

## Abstract

In this paper, an optimal control problem for a novel nonlinear mathematical model with time-delay for cancer treatment based on synergy between anti-angiogenic and immune cell therapies is presented. The governed model consists of eighteen fractional-order delay differential equations, where the fractional derivative is defined in the Caputo sense. A discrete time-delay is incorporated to represent the time required for immune system to interact with tumor, and fractional-order derivative is considered to reflect the memory and hereditary properties in the process. Two control variables for immunotherapy and anti-angiogenic therapy are considered to reduce the load of cancer cells. Necessary and sufficient conditions that guarantee the existence and the uniqueness for the solution of the control problem have been considered. Some numerical simulations are given to validate the theoretical results.

# Stability analysis of multi-valued neural networks with time delays

R. Rakkiyappan<sup>1</sup>

<sup>1</sup> Department of Mathematics, Bharathiar University, Tamilnadu, India  
Email: rakkigru@gmail.com

## Abstract

Neural networks with values in multidimensional domains have been intensively studied over the last few years, because of their immense potentials in application perspective. For a complex-valued neural networks(CVNNs), the main challenge is the choice of activation functions. In a real-valued neural networks(RVNNs), the node activation functions is usually chosen to be a smooth, bounded and non constant function. There is no problem in selecting a real valued functions that satisfies these type of requirements. In a CVNNs any regular analytic function cannot be bounded unless it reduces to a constant. That is to say, the activations in CVNNs cannot be both bounded and analytic. Therefore, the activations are main challenge for CVNNs. Quaternion does not meet commutative law with respect to multiplication, so the quaternion is much harder than that of other networks like RVNNs, CVNNs. Quaternion-valued neural networks (QVNNs) generic extension of RVNNs or CVNNs. More recently, the octonion-valued neural networks(OVNNs) gains its attention since the octonian algebra is a different generalization of the complex and quaternion algebra with an 8-dimensional normed division algebra that can be defined over the field of real numbers. To avoid the non associativity of the octonions and the non commutativity of the quaternion, the OVNNs model is decomposed into four complex-valued systems, using the Cayley-Dickson construction method. The existence of time delays frequently causes oscilation, divergence in neural networks. Since the time delay dependent results are looser than the time delay independent ones when the delay are small. So it is necessary to propose the time delay dependent stability results. For different classes of activation functions, different approaches are necessary to analyze the relevant neural networks. Some new sufficient delay-dependent stability criteria are established to ensure the stability of equilibrium point for the addressed neural networks by constructing an appropriate Lyapunov-Krasovskii functionals(LKFs). The obtained stability criteria are established in the form of LMIs. Finally, numerical simulation results are provided to illustrate the validity of the theoretical results.

# Characteristic functions of oscillatory functional differential equations of mixed type

Md.Abdul Malique<sup>1</sup>

<sup>1</sup>Department of Mathematics, University of Hafr Al Batin, KSA

Email: amalique@uohb.edu.sa

## Abstract

In this paper we study characteristic functions of linear homogeneous functional differential equations of mixed type. Our interest is in the qualitative properties of characteristics functions of functional differential equations of mixed type. The motivation of the work is to study oscillatory and non-oscillatory solutions of functional differential equations. Characteristic function is a key tool of an oscillatory or non-oscillatory solution. We shall consider a basic differential equations of mixed type of the form

$$y'(t) = \sum_{k=-1, j=1}^{1,3} b_j y(t+k),$$

where  $y(t) \in \mathcal{R}$ ,  $k \in \mathcal{R}$ ,  $k \in [-1, 1]$ ,  $b \in \mathcal{R}$ ,  $b > 0$ ,  $j > 0$ ,  $j \in [1, 3]$ . We conclude with our study and further work.

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# Existence of Periodic Solutions for the Discrete-Time Counterpart of a Neutral-Type Cellular Neural Network with Time -Varying Delays and Impulses

H. Akca<sup>1</sup> and V. Covachev<sup>2\*</sup>

<sup>1</sup> Department of Applied Sciences and Mathematics, Collage of Arts and Sciences, Abu Dhabi University, Abu Dhabi, UAE

<sup>2</sup> Department of Mathematical Sciences, Bulgarian Academy of Science, Sofia, Bulgaria

haydar.akca@adu.ac.ae (\* Corresponding author: Haydar Akca )

## Abstract

From the mathematical point of view, a cellular neural network (CNN) can be characterized by an array of identical nonlinear dynamical systems called cells (neurons) that are locally interconnected. Using the semi-discretization method, in the present paper a discrete-time counterpart of a neutral-type CNN with time-varying delays and impulses is constructed. Sufficient conditions for the existence of periodic solutions of the discrete-time system thus obtained are found by using the continuation theorem of coincidence degree theory. Over the past two decades neural networks have been widely studied since they have been successfully applied to various processing problems such as optimization, image processing, associative memory and many other fields. Different types of applications depend on the dynamical behaviors of the neural networks. Cellular neural networks (CNNs) were introduced in the 1980s by Chua and Yang. Since then, many researchers have done extensive and interesting works on this subject because of its potential applications in real-life problems such as signal processing, pattern recognition, chemical processes, nuclear reactors, biological systems, static image processing, associative memories, optimization problems and so on.

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# Oscillation criteria for second-order quasi-linear neutral functional differential equation

Osama Mooaz<sup>1</sup> and Omar Bazighifan<sup>2</sup>

<sup>1</sup>Department of Mathematics, Faculty of Science, Mansoura University, Mansoura, 35516, Egypt

<sup>2</sup>Department of Mathematics, Faculty of Education, Hadhramout University, Yemen

Email: o\_moaaz@mans.edu.eg

## Abstract

In this work, new sufficient conditions for oscillation of second-order quasi-linear neutral functional differential equation are established. In numerous applications in biology, electrical engineering or physiology, the dependence on the past appears naturally. Differential equations with delay appears in modeling of these natural phenomena. One objective of our paper is to further simplify and complement some well-known results which were published recently in the literature. Firstly, we improve results of Liu et al. (2012) so that the condition guarantee that all solutions are oscillatory, and without imposing restrictions on the derivatives of  $p(t)$ ,  $\tau(t)$ . As well, we simplify results of Saker [2010] and Wu et al. (2016) by obtaining criteria ensure oscillation without verifying the extra conditions. In order to support our results, we introduce illustrating example.

## References

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# Qualitative analysis of delayed SIR epidemic model with a saturated incidence rate

Mohamed-Naim Y. Anwar<sup>1</sup>, Hebatallah J Alsakaji<sup>2</sup>, Fathalla A. Rihan<sup>2</sup>

<sup>1</sup> Faculty of Engineering, Pharos University in Alexandria, Egypt

<sup>2</sup>Department of Mathematical Sciences, College of Science, UAE University, 15551, Al Ain,  
UAE

Email: mohamed.naeem@pua.edu.eg

## Abstract

This work is concerned a delayed SIR epidemic model in which the susceptibles are assumed to satisfy the logistic equation and the incidence term is of saturated form with the susceptible. We investigate the qualitative behaviour of the model and find the conditions that guarantee the asymptotic stability of corresponding steady states. We present the conditions in the time lag in which the DDE model is stable. Hopf bifurcation analysis is also addressed. Numerical simulations are provided in order to illustrate the theoretical results and gain further insight into the behaviour of this system.

# Partial differential equations with delay term

M. Yebdri<sup>1\*</sup> and H. Harraga<sup>1</sup>

<sup>1</sup> Department of Mathematics, University of Tlemcenn, Tlemcen, Algeria

m\_yebdri@mail.univ-tlemcen.dz (\* Corresponding author: Mustapha Yebdri)

## Abstract

The theory of differential equations with delay occupies an important place in the development of the theory of infinite dimensional dynamical systems. This theory covers ordinary and partial differential equations with delay ([1, 5]). Comparing to functional differential equations, there are a few works dealing with partial differential equations with delay (see [2, 4, 3, 5]). Even if the theory of functional analysis (see [1, 2]) allow us to study a wide class of partial differential equations, the literature sources of fundamental facts and approaches for partial differential equations with delay term are very few [1, 2, 3].

In this talk we establish the existence of weak solutions to a class of partial differential equation with delay which involve a nonlinear delay. For this one proceed by the compactness method i.e. we construct approximate solutions by reduction to the finite dimensional, for example by Galerkin method. So we obtain the existence of approximate solutions using an existence theorem of solution of a system of ordinary differential equations. After, one go to the limit on the dimension (see [3]).

## References

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# Stability and numerical simulations for an SIR epidemic model with distributed delay and vaccination

A. ELAZZOUZI<sup>1\*</sup>, A. LAMRANI ALAOU<sup>2</sup> and A. M. TILIOUA<sup>2</sup>

<sup>1</sup> Department of MPI, University of Sidi Mouhamed Ben Abdellah, FP Taza, LSI Laboratory, Morocco

<sup>2</sup> Department of Mathematics, University of Moulay Ismaïl, FST Errachidia, M2I Laboratory, MAMCS Group, Morocco

abdelhai.elazzouzi@usmba.ac.ma (\* Corresponding author: A. ELAZZOUZI)

## Abstract

This work is concerned with the global dynamics of an SIR epidemic model with a wide class of generalized nonlinear incidence rate, distributed delay and vaccination. By constructing a Lyapunov functionals, we show that the disease free equilibrium state is globally asymptotically stable when the basic reproduction number  $R_0$  is less than or equal to one and the disease endemic equilibrium is globally asymptotically stable when  $R_0$  is greater than one. We provide a numerical simulations to illustrate the effect of vaccination on the model. It is shown, when the basic reproduction number for vaccination-free model  $R_0 > 1$  that the disease of the SIR model proposed in [3] can be controlled but it can not be eradicated. Here, in this work, we give an example in which for certain values of the vaccination parameter  $p$  the disease will die out even if  $R_0 > 1$ . In other side, we compare numerical results of the distributed delay model and the discrete time delay model.

## References

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# Effect of CTL and antibody immune responses on the virus dynamics with both virus-to-cell and cell-to-cell transmissions and Holling type-II

Ahmed M. Elaiw and Shafeek A. Ghaleb

<sup>1</sup> Department of Mathematics, Faculty of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia

E-mail: shafeekye2@gmail.com

## Abstract

Mathematical modeling and analysis of viral infection models have attracted the interest of several researchers during the last decades. These works can help researchers for better understanding the viral dynamical behavior and providing new suggestions for clinical treatment. The basic viral infection model presented in [1] has focused on modeling the interaction between three main compartments, uninfected cells, infected cells and free virus particles. Cytotoxic T Lymphocyte (CTL) cells and antibodies play central role of controlling the viral progression. CTL cells attack and kill the infected cells. The B cell produces antibodies to neutralize the viruses. Wodarz [2] has presented a viral infection model with both CTL and antibody immune responses. The model presented in [2] is based on the assumption that the uninfected cells becomes infected by contacting with the virus (virus-to-cell transmission). However, it has been reported in [3] that the uninfected cells can also become infected when it contacts an infected cell (cell-to-cell transmission).

In this paper, we propose and analyze the effect of CTL and antibody immune responses on the virus dynamics with both virus-to-cell and cell-to-cell transmissions. The infection rate is given by Holling-type incidence. We first show that the model is biologically acceptable by showing that the solutions of the model are nonnegative and bounded. We find the steady states of the model and investigate their global stability analysis. We derive five threshold parameters which fully determine the existence and stability of the five steady states of the model. The global stability of all steady states of the models is proven using Lyapunov method and applying LaSalle's invariance principle, we established the global asymptotic stability of the steady states of the model. To support our theoretical results we have performed some numerical simulations for the model. The results show the effect of the CTL and antibody immune response in controlling the disease and in removing the virus from the body.

## References

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# Global stability of latent Chikungunya virus dynamics model with multitarget cells and discrete time delays

Ahmed M. Elaiw, Taofeek O. Alade and Saud M. Alsulami

<sup>1</sup> Department of Mathematics, Faculty of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia

E-mail: taofeekalade@gmail.com

## Abstract

Chikungunya virus (CHIKV) is an alphavirus which is transmitted to humans by *Aedes aegypti* and *Aedes albopictus* mosquitoes. The CHIKV attacks the target cells and causes Chikungunya fever with symptoms such as rashes, headache and fever. Symptoms usually occur after an incubation period of 4-7 days. CHIKV has been an outbreak in the last decade in some parts of the world and has increased the population of people at risk [1]. Although CHIKV pathogenesis in humans is not yet completely understood, but recent outbreaks have provided deep understanding into the organs and cells involved in the viral replication. CHIKV infects and replicates in a variety of cells, such as fibroblasts, macrophages, monocytes, skeletal muscle satellite cells and other skin cells [2]. In the literature of CHIKV infection, most of the mathematical models presented described the disease transmission in mosquito and human populations [3-5]. This paper proposes a latent Chikungunya viral infection model with multitarget cells. The model incorporates (i) two types of infected cells, latently infected cells and actively infected cells which produce the CHIKV particles (ii) saturated incidence rate which is suitable to model the nonlinear dynamics of the CHIKV especially when its concentration is high (iii) antibody immune response. The model is an  $(3n+2)$ -dimensional system of nonlinear delay differential equations (DDEs) that describes the population dynamics of CHIKV,  $n$  categories of uninfected target cells,  $n$  categories of latently and actively infected cells. The model is incorporated by intracellular discrete time delays. The qualitative behavior of the model is studied. Using the method of Lyapunov function, we established the global stability of the equilibria of the model. The effect of the time delay on the stability of the equilibria has also been illustrated by numerical simulations.

**Keywords:** Global stability; Chikungunya virus infection; Time delays; Multitarget cells; Lyapunov function

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# Release of Breeding Suppression Advanced Time Delay Predator-Prey Dynamics

Qamar, J. A. Khan and Fatma A. Al-kharousi

Department of Mathematics, Sultan Qaboos University, Muscat, Oman

E-mail: qjalil@squ.edu.om

## Abstract

This paper present a theoretical model where prey, due to short term heavy predation pressure, temporarily suppress breeding and restart breeding when in future predation pressure lowers. The predator consumes both suppressor and breeder of the prey and this prey population is more prone to predation at higher densities. Equilibrium and stability analysis are carried out. Taking fraction of new born breeder prey,  $q$  as a bifurcation parameter it is shown that Hopf bifurcation could occur.

**Keywords:** Prey-predator, Time delay model, Equilibrium points, Stability analysis, Hopf bifurcation

# Dynamics of visceral leishmaniasis model with time delays

Velmurugan Gandhi, Ibrahim Elmojtaba and Nasser Al Salti

Department of Mathematics, Sultan Qaboos University, Muscat, Oman

E-mail: velmurugan@squ.edu.om

## Abstract

In this work, authors discuss some of the dynamical characteristics of visceral leishmaniasis model with time delays. First, we propose a mathematical model for visceral leishmaniasis disease with time delay. Then, by analyzing the characteristic equations, some sufficient conditions are derived to ensure the stability of equilibrium points (disease-free equilibrium and endemic equilibrium) for the considered delayed visceral leishmaniasis model. Moreover, the time delay is taken as bifurcation parameter and establish some sufficient criteria to show the existence of Hopf bifurcation of the considered model. Finally, some numerical simulations are given to show the effectiveness of our theoretical results.

**Key words:** Endemic diseases, Leishmaniasis model with delay, Equilibrium points, Stability analysis, Bifurcation analysis, Numerical simulations.

## SIR Model with Time-Varying Contact Rate

Fatma Al-Musalhi, Nasser Al-Salti, Ibrahim Elmojtaba, Velmurugan Gandhi

Department of Mathematics, Sultan Qaboos University, Muscat, Oman

E-mail: Fatma.almusalhi@outlook.com

### Abstract

The contact rate is defined as the average number of contacts adequate for disease transmission by an individual per unit time and it is almost always assumed to be constant in time. However, in reality, the contact rate is not always constant throughout the year due to different factors such as population behaviour, environmental factors and many others. Therefore, it is more realistic to consider it to be a function of time.

In this talk, an SIR model with time-varying contact rate is considered. The existence and local stability of the equilibria of the model are analyzed. Results on global stability of disease-free equilibrium and transcritical bifurcation are proved. Finally, numerical simulations are presented to illustrate the theoretical results and to demonstrate the effect of the model parameters related to the contact rate on the behaviour of the solutions.

**Key words:** Endemic diseases, SIR model, Equilibrium points, Bifurcation analysis, Global stability, Numerical simulations.

# Mathematical Analysis for the Dynamics of an Enteric Disease Model

Shaima Al Shanfari, Ibrahim M. Elmojtaba and Nasser Al Salti

Department of Mathematics, Sultan Qaboos University, Muscat, Oman

E-mail: s53774@student.squ.edu.om

## Abstract

Enteric diseases range from mild food poisoning to dangerous infections like typhoid fever and cholera. Enteric and diarrheal diseases kill about 500,000 children under 5 each year, and those who survive face repeated infections by gut pathogens in the early years of their life, which can lead to serious, lifelong health problems. There are many challenges associated with modelling of enteric diseases. However, the most important are their indirect transmission mode and the role of mechanical vectors in the transmission circle. The aim of this research is to propose a new model to overcome those challenges. The model will be analyzed and all of its properties will be shown both analytically and numerically.

**Key words:** Enteric disease, Equilibrium points, Bifurcation analysis, Global stability, Optimal control, Numerical simulations

# Prey-Predator Models with Variable carrying capacity

Maryam Al-Maqbali, Nasser Al-Salti and Ibrahim Elmojtaba

Department of Mathematics, Sultan Qaboos University, Muscat, Oman

E-mail: maryamalmoqbalix@gmail.com

## Abstract

Prey-predator models with variable carrying capacity are proposed. These models are more realistic in modelling population dynamics in an environment that undergoes changes. In particular, prey-predator models with Holling type I and type II functional responses, incorporating the idea of a variable carrying capacity are considered. The carrying capacity was modelled by a logistic equation that increases sigmoidally between an initial value  $k_0$ , i.e. the initial value of the carrying capacity, which lies between a lower value  $k_1$  and a final value  $k_1 + k_2$ , where  $k_2$  is the range on which the carrying capacity is changing. In order to examine the effect of the variable carrying capacity on the prey-predator dynamics, the two models were analyzed qualitatively using stability analysis and numerical solutions for the prey and the predator population densities were also obtained. Results on global stability and Hopf bifurcation of certain equilibrium points have been also presented. The effect of other model parameters on the prey-predator dynamics has been also examined. In particular, results on the effect of handling parameter and the predator's death rate, which has been to be the bifurcation parameter were presented.

**Key words:** Prey-predator, variable carrying capacity, Limit cycle, Hopf bifurcation



# Controllability analysis of nonlinear fractional order differential systems with state delay and non-instantaneous impulsive effects

B.Sundaravadivoo

Department of Mathematics, Alagappa University, Karaikudi-630 004, India

E-mail: [sundaravadivoo@gmail.com](mailto:sundaravadivoo@gmail.com)

## Abstract

This manuscript prospects the controllability criteria of non-instantaneous impulsive nonlinear fractional differential systems with state delay. By enrolling an appropriate Gramian matrix which is often defined by the Mittag-Leffer function and with the assistance of Laplace transform, the conditions to obtain the necessary and sufficient for the controllability of non-instantaneous impulsive fractional differential systems with state delay are derived using algebraic approach. An important feature presents in the paper is that, taken non-instantaneous impulses into the fractional order dynamical system with state delay and studied the controllability analysis, since this not exists in the available source of literature. Inclusively, two illustrative examples are provided with the existence of state delay non-instantaneous impulse fractional dynamical system. So this demonstrates the validity and efficacy of the obtained criteria of the main section.

# On the asymptotic stability and boundedness of solutions stochastic differential equations with multiple time lags

Cemil Tunç<sup>1\*</sup> and Osman Tunç<sup>2</sup>

<sup>1\* 2</sup> Department of Mathematics, Faculty of Sciences, Van Yuzuncu Yil University 65080,  
Van-Turkey

cemtunc@yahoo.com (\* Corresponding author: Cemil Tunç)

## Abstract

In this paper, we consider a class of stochastic non-linear differential equations of second order with multiple delays. We establish new sufficient conditions which guarantee stochastic stability, stochastic asymptotic stability of the zero solutions and boundedness of all solutions of the considered stochastic delay differential equations. The technique of the proof is based on the construction of suitable Lyapunov functionals. The results of this paper extend and improve some recent results that can be found literature on the subject, and they are new and have novelty. As an application of the obtained results, in the particular case, an example is given for illustrations by using MATLAB-Simulink.

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# On the uniform asymptotic stability, integrability and boundedness of solutions of Volterra integro-differential equations with infinite delay

Osman Tunç<sup>1</sup> and Cemil Tunç<sup>2\*</sup>

<sup>2</sup> Department of Mathematics, Faculty of Sciences, Van Yuzuncu Yil University, 65080,  
Van-Turkey

cemtunc@yahoo.com (\* Corresponding author: Cemil Tunç)

## Abstract

In this paper, two new Lyapunov functionals are defined. We apply that functionals to get sufficient conditions guaranteeing the asymptotic stability, uniform stability and boundedness of solutions of certain non-linear Volterra integro-differential equations of first order. The results obtained have improvements and extensions of the former the results that can found in literature. We give examples to show applicability of the results obtained and for illustrations. In the particular cases, using MATLAB-Simulink, it is clearly shown the behaviors of the orbits of the Volterra integro-differential equations considered.

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# Stability results and existence theorems for nonlinear delay-fractional differential equations with $\Phi_p$ -operator

Hasib Khan<sup>1,2</sup>, Cemil Tunç<sup>3</sup>, Aziz Khan<sup>4</sup>

<sup>1</sup> College of Engineering Mechanics, Hohai University, Jiangsu, 211100, Nanjing PR., China

<sup>2</sup> Department of Mathematics, Shaheed Benazir Bhutto University, Dir Upper, 18000, Khybar Pakhtunkhwa, Pakistan

<sup>3</sup> Department of Mathematics, Faculty of Sciences, Van Yuzuncu Yil University, 65080, Van-Turkey

<sup>4</sup> Department of Mathematics, University of Peshawar, 25000, Peshawar, Khybar Pakhtunkhwa, Pakistan

cemtunc@yahoo.com

## Abstract

The study about delay-fractional differential equations (fractional DEs) have recently been got a great attention of scientists in many different subjects based on mathematically modeling. In the study of fractional DEs the first question raises is whether the problem will have a solution or not. Also, whether the problem is stable or not? In order to ensure the answer of these questions, we study the study existence and uniqueness of solutions (EUS) as well Hyers-Ulam stability for our proposed problem a nonlinear fractional DE with a an operator  $\Phi_p$  and a non zero delay  $\tau > 0$  in Banach space  $\mathcal{Y}$  of order  $n - 1 < \sigma, \beta \leq n, n \geq 3$ , in the Caputo's sense. The assumed singular fractional DE with  $\Phi_p$ -operator is more general and complex than studied for stabilities by [3].

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# Bifurcation Analysis of a Delay differential Model of Predator-Prey System with Allee Effect

Hebatallah J. AlSakaji<sup>1</sup> and Fathalla A. Rihan<sup>1</sup>

<sup>1</sup> Department of Mathematical Sciences, College of Science, UAE University, Al Ain, UAE

heba.sakaji@uaeu.ac.ae & frihan@uaeu.ac.ae

## Abstract

In recent years, a large number of mathematical models that described by delay differential equations (DDEs) have appeared in the life sciences. In this work, we discuss the dynamics and bifurcation analysis of three species delayed predator-prey model with competition among preys. The growth of both prey populations is subjected to the Allee effect in the logistic terms (the growth rate increases with the population density while it decreases at larger densities), and with discrete time-delay in the reaction terms. We study the existence of positive equilibrium points and their local and global asymptotic stability results. Bifurcation analysis in terms of time delay is investigated. The stable, periodic and chaotic behaviours of the model depending on parameter values. The presence of time-delay in the model improves the stability of the solutions and enriches the dynamics of the model. Numerical simulations are provided to illustrate the theoretical results.

**Keywords:** Allee effect; Bifurcation analysis; Functional response; Predator-prey model; Time-delay

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# MINIMUM ENERGY COMPENSATION FOR DISCRETE DELAYED SYSTEMS WITH DISTURBANCES

SALMA SOUHAILE AND LARBI AFIFI

Faculty of Sciences Ain Chock, University Hassan II, B.P.5366, Maârif, Casablanca, Morocco  
souhaile.salma@gmail.com

## Abstract

This work is devoted to the remediability problem for a class of discrete delayed systems. We investigate the possibility of reducing the disturbanc effect with a convenient choice of the control operator. We give the main properties and characterization results of this concept, according to the delay and the observation. Then, under appropriate hypothesis, we demonstrate how to find the optimal control which ensures the compensation of a disturbance measured through the observation (measurements, signals,...). The discrete version of the wave equation as well as the usual actuators and sensors are examined. Numerical results are also presented.

# A Reliable Method for First Order Delay Equation Based on the Implicit Hybrid Method

Mohamed I Syam

Department of Mathematical Sciences, College of Science, UAE Univeraity, al Ain 15551, UAE

Email: m.syam@uaeu.ac.ae

## Abstract

In this paper, a reliable method for solving first order delay problem based on the implicit hybrid method. Theoretical results are investigated. Some of our numerical examples are presented to show the efficiency of the proposed method.

## 6 List of Participants

Name	Email	Affiliation
Md . Abdul Malique, 11	amalique@uohb.edu.sa	Univ. of Hafr Al Batin, Saudi Arabia
Ephraim Agyingi, 8	eoasma@rit.edu	Rochester University, USA
Haydar Akca, 12	Haydar.Akca@adu.ac.ae	Abu Dhabi University, UAE
Hebatallah J. Alsakaji, 28	heba.sakaji@uaeu.ac	UAE University, UAE
Nasser Al-Salti, 20,21,22	nalsalti@squ.edu.om	SQU, Oman
Shima Al-Shanfari, 22	s53774@student.squ.edu.om	SQU, Oman
Maryam Al Maqbali, 23	maryamalmoqbalix@gmail.com	SQU, Oman
Maryam Al Yahyai	alyahyai44@hotmail.com	SQU, Oman
Mohamed N. Anwar, 14	mohamed.naeem@pua.edu.eg	Pharos University, Egypt
Fatma Al-Musalhi	Fatma.almusalhi@outlook.com	SQU, Oman
Seham Almekhlafi, 9	smdk100@gmail.com	Sana'a University, Yemen
Taofeek Alade, 18	taofeekalade@gmail.com	King Abdulaziz Univ., Saudi Arabia
Gennady Bocharov, 3	bocharov@m.inm.ras.ru	Russian Academy of Sciences, Russia
Omar Bazighifan, 13	o.bazighifan@gmail.com	Hadhramout University, Yemen
Ibrahim Elmojtaba, 20,21	elmojtaba@squ.edu.om	SQU, Oman
Abdelhai ELAZZOUZI	abdelhai.elazzouzi@usmba.ac.ma	University of Moulay Ismail, Morocco
Velmurugan Gandhi,20	velmurugan@squ.edu.om	SQU, Oman
Shafeek A. Ghaleb,17	shafeekye2@gmail.com	King Abdulaziz Univ., Saudi Arabia
Qamar Khan,19	qjalil@squ.edu.om	SQU, Oman
Yang Kuang, 4	kuang@asu.edu	Arizona State University, USA
Osama Moaaz,13	o_moaaz@mans.edu.eg	Mansoura University, Egypt
Rajan Rakkiyappan, 10,	rakkigr@gmail.com	Bharathiar University, India
Fathalla A. Rihan, 6	frihan@uaeu.ac.ae	UAE University, UAE
Nasser Sweilam, 9	nsweilam@sci.cu.edu.eg	Cairo University, Egypt
Salma Souhaile, 29	souhaile.salma@gmail.com	University Hassan II, Morocco
B. Sundaravadivoo, 24	sundaravadivoon@gmail.com	Alagappa University, India
Hongjiong Tian, 7	hjtian@shnu.edu.cn	Shanghai Normal University, China
Cemil Tunc, 25,26,27	cemtunc@yahoo.com	Yuzuncu Yil University Turkey
Mohammad I. Sysm, 30	m.syam@uaeu.ac.ae	UAE University
Radouane Yafia, 5	r.yafia@uiz.ac.ma	Ibn Zohr University, Morocco
Mustafa Yebrdi, 15	yebdri@yahoo.com	Abou Bakr Belkaid Univ., Algeria