# Delay Differential Equations: Theory, Applications and New Trends (DDEs-TANTs 2018)

Workshop from 3–4 Oct., 2018, United Arab Emirates University, Al-Ain, UAE

# Book of Abstracts


UNITED ARAB EMIRATES UNIVERSITY

College of Science
Department of Mathematical Sciences
UAE
3–4 October, 2018

## Introduction

Recently, there has been increasing interest related to the theory of delay differential equations (DDEs), due to their important applications in physics, biology, ecology, and physiology. The main objective of this well-focused workshop (DDEs-TANTs 18) is to provide an opportunity to study the new trends and analytical insights of the delay differential equations, existence and uniqueness of the solutions, boundedness and persistence, oscillatory behavior of the solutions, stability and bifurcation analysis, parameter estimations and sensitivity analysis, and numerical investigations of solutions. Potential topics include but are not limited to the following: (i) Development of novel theories or improvement to existing theories on delay differential equations; (ii) Development of novel numerical approaches for delay differential equations; (iii) Stability, sensitivity analysis and bifurcation analysis of time delay models; (iv) Optimal control of nonlinear delay differential systems.

The scientific program of the workshop consists of 50-minute plenary lectures, 30-minute invited lectures and 20-minute contributed talks.

The selected papers presented at DDEs-TANRs workshop will be published in a special issue of the journal "Discrete & Continuous Dynamical Systems - S (DCDS-S)", one of the American Institute of Mathematical Series. This workshop and special issue is dedicated to the memory of Professor Christopher T. H. Baker (1939–2017).

## Date and Venue:

October 3–4, 2018, United Arab Emirates University Al Ain, UAE.

## Keynote Speakers:

1. Prof. Gennady Bocharov, Institute of Numerical Mathematics, Russian Academy of Sciences, Moscow;
2. Prof. Yang Kuang, School of Math and Statistical Sciences, Arizona State University, USA;
3. Prof. Radouane Yafia, Ibn Zohr University, Morocco.

## Organizing Committee:

1. Prof. Fathalla A. Rihan (Chair, UAEU), Email: frihan@uaeu.ac.ae
2. Dr. Nasser Al-Salti (SQU, OMAN)
3. Prof. Mohamed Syam (UAEU)
4. Dr. Abdessamad Tridane (UAEU)

 ISBN: XXX-XXX-XXXX-X-X

## Scientific Committee:

1. Prof. Fathalla A. Rihan (UAE)
2. Prof. G. Bocharov (Russia)
3. Prof. Yang Kuang (USA)
4. Prof Hongjiong Tian (China)
5. Prof. Haydar Akca (UAEU)
6. Prof. Ephraim Agyingi (USA)
7. Prof. Radouane Yafia (Morocco)
8. Prof. Nasser Sweilam (Egypt)
9. Dr. I. M. Elmojtaba (Oman)
10. Prof. Cemil Tunc (Turky)

## Logistic Committee:

1. Dr Youssef El-Khatib
2. Dr. Sehjeong Kim
3. Dr. Muhammad Imran
4. Ms. Hebatallah J. Alsakaji


ISBN: XXX-XXX-XXXX-X-X

# Delay differential equations for modelling and control of virus infection dynamics

G. Bocharov[1]

[1] Marchuk Institute of Numerical Mathematics, Russian Academy of Sciences, Moscow, Russian Federation

Email: bocharov@m.inm.ras.ru

**Abstract**

Systems of delay differential equations (DDE) are widely used for studying the dynamics of virus infections and immune responses as individual models or building blocks of hybrid multi-scale models [1]. Virus diseases are characterized by a variety of courses and outcomes. Understanding the mechanisms of chronic infections and elaborating approaches to their treatment is central to mathematical immunology. Effective treatment of infections (e.g., HIV) requires application of multiple drugs acting on the virus- and the host organism physiology. Earlier, for the DDE model of infectious disease Guri I. Marchuk proposed an approach to cure the chronic infections via perturbing the respective steady states. We present a computational approach to designing multi-modal perturbations using the so called, optimal disturbances [2]. The optimal disturbances of the steady states are computed by maximizing the perturbation-induced response. The control approach is illustrated for a multi-stable DDE system modeling the experimental LCMV infection. The optimal disturbances of high viral load steady state driving it to a low viral load steady state are computed. Comparison of the optimal disturbances obtained using either the $L_2$- or $W_2^1$ norm is presented [3]. It is argued that optimal disturbances computed using the Sobolev norm are superior for designing multi-modal therapies.

The research was funded by the Russian Science Foundation (Grant no. 18-11-00171).

## References

[1] Gennady Bocharov, Vitaly Volpert, Burkhard Ludewig, Andreas Meyerhans *Mathematical Immunology of Virus Infections*, Springer, Cham, 2018.

[2] Gennady Bocharov, Yuri Nechepurenko, Michael Khristichenko Dmitry Grebennikov *Maximum response perturbation-based control of virus infection model with time-delays* , Russ. J. Numer. Anal. Math. Modelling, 32:5, 275-291, 2017.

[3] Gennady Bocharov, Yuri Nechepurenko, Michael Khristichenko, Dmitry Grebennikov *Optimal Disturbances of Bistable Time-Delay Systems Modeling Virus Infections*, Doklady Mathematics, 98:1, 1-4, 2018 (in press).

 ISBN: XXX-XXX-XXXX-X-X

# Oscillatory dynamics of an intravenous glucose tolerance test model with delay interval

Y. Kuang[1]

[1]School of Mathematical and Statistical Sciences, Arizona State University, Tempe, Arizona, USA

Email: kuang@asu.edu

**Abstract**

Type 2 diabetes mellitus (T2DM) has become prevalent pandemic disease largely due to the modern life style. The intravenous glucose tolerance test (IVGTT) is an effective protocol to determine the insulin sensitivity, glucose effectiveness and pancreatic -cell functionality, through the analysis and parameter estimation of a proper differential equation model. In this talk we propose a novel approach to model the time delay in IVGTT modeling. This approach uses two parameters to simulate not only both discrete time delay and distributed time delay in the past interval, but also the time delay distributed in a past sub-interval. Longer time delay, either a discrete or distributed delay, often destabilize a system. This may not be true for time delay over a sub-interval. We present analytically some basic model properties which are desirable biologically and mathematically. We show that this relatively simple model provides good fit to fluctuating patient data sets and reveals some intriguing dynamics. Moreover, our numerical simulation results indicate that our model may remove the defect in well known Minimal Model which often overestimates the glucose effectiveness index.

## References

[1] Xiangyun Shi, Yang Kuang, Athena Makroglou, Sriprakash Mokshagundam, and Jiaxu Li Oscillatory dynamics of an intravenous glucose tolerance test model with delay interval, Chaos, 27, 114324, 2017.

 ISBN: XXX-XXX-XXXX-X-X

# Hopf Bifurcation in Delay Differential Equations and Applications

R. Yafia[1]

[1]Department of Mathematics, Ibn Zohr University, CST Ait Melloul, Agadir, Morocco
Email: r.yafia@uiz.ac.ma

**Abstract**

This talk is concerned with the Hopf-bifurcation phenomenon in delayed differential equations, which occurs naturally in biology, ecology and physics etc. Based on the reduction of the abstract delay differential equation to a two-dimensional system via the variation of constant formula and the center manifold Theorem and estimating the distance between solutions of the original equation and the bifurcating periodic solutions, we estimate the stability region of bifurcating branch. Considering time delay as a parameter of bifurcation in a delayed system of differential equations which modeling the interaction between two species in population dynamics. We analyze the local stability/instability of the possible equilibrium points and the occurrence of Hopf bifurcation for some critical value of time delay. In the end, we prove the stability/instability of the bifurcating periodic solutions via normal form Theory and center manifold Theorem.

## References

[1] Diekmann O., Van Giles S., Verduyn Lunel S. andWalter H., Delay equations. Springer-Verlag, New-York (1995).

[2] T. Faria et M. L. Magalhaes, Normal Form for Retarded Functional Differential Equations with Parameters and Applications to Hopf Bifurcation, J. D. E. 122 (1995) 181-200.

[3] J.K.Hale and S. M. Verduyn Lunel, Introduction to functional Differential equations. Springer-Verlag, New-York (1993).

[4] B. D. Hassard, N. D. Kazarinoff et Y. H.Wan, Theory and Application of Hopf Bifurcation, Combridge University Press, London Mathematical Society, Lecture notes Series 41 (1981).

 ISBN: XXX-XXX-XXXX-X-X

# Qualitative and Quantitative Features of Delay Differential Equations in Biosciences

Fathalla A. Rihan[1]

[1]Department of Mathematical Sciences, College of Science, UAE University, 15551, Al Ain, UAE
Email: frihan@uaeu.ac.ae

**Abstract**

Delay differential equations (DDEs) are a class of differential equations that have received considerable attention and been shown to model many real life problems, traditionally formulated by systems of ordinary differential equations (ODEs), more naturally and more accurately. Such class of DDEs are widely used for analysis and predictions of systems with memory such as population dynamics, epidemiology, immunology, physiology, neural networks and other biological and physical systems with memory. The aim of this talk is to provide a wide range of DDEs and show that they have a richer mathematical framework for the analysis of dynamical systems. We present suitable numerical techniques for solving the forward and backward problems of DDes. Sensitivity analysis is also investigated for DDEs.

# References

[1] G.A. Bocharov & F.A. Rihan, Numerical modelling in biosciences using delay differential equations, *Journal of Computational and Applied Mathematics* **125** (2000) 183–199.

[2] C.T.H Baker, G. Bocharov, E. Parmuzin, F.A. Rihan, Some aspects of causal & neutral equations used in modelling, *Journal of Computational and Applied Mathematics* **229** (2009) 335–349.

[3] F. A. Rihan, C. Tunc, S. H. Saker, S. Lakshmanan, and R. Rakkiyappan, Applications of Delay Differential Equations in Biological Systems, Complexity (Editorial) 2018, 1?3.

[4] F. A. Rihan, A. Azamov and H. J. Al-Sakaji, An Inverse Problem For Delay Differential Equations: Parameter Estimation, Nonlinearity, Sensitivity, Applied Mathematics & Information Sciences, 12 (1) (2018), 63?74.

[5] F. A. Rihan, N. F. Rihan, Dynamics of Cancer-Immune System with External Treatment and Optimal Control, Journal of Cancer Science & Therapy, 8(10), (2016), 257- 261

 ISBN: XXX-XXX-XXXX-X-X

# Delay Differential Model with Optimal Control for a Cancer Treatment Based on Synergy Between Anti-angiogenic and Immune Cell Therapies

N.H. Sweilam[1], F.A. Rihan[2], S. M. El-Mekhlafi[3]

[1]Department of Mathematics, Faculty of Science, Cairo University, Egypt
[2]Department of Mathematical Sciences, College of Science, UAE University, 15551, Al Ain, UAE
[3]epartment of Mathematics, Faculty of Education, Sana'a University, Yemen
Email: nsweilam@sci.cu.edu.eg

**Abstract**

In this paper, an optimal control problem for a novel nonlinear mathematical model with time-delay for cancer treatment based on synergy between anti- angiogenic and immune cell therapies is presented. The governed model consists of eighteen fractional-order delay differential equations, where the fractional derivative is defined in the Caputo sense. A discrete time-delay is incorporated to represent the time required for immune system to interact with tumor, and fractional-order derivative is consider to reflect the memory and hereditary properties in the process. Two control variables for immunotherapy and anti-angiogenic therapy are considered to reduce the load of cancer cells. Necessary and sufficient conditions that guarantee the existence and the uniqueness for the solution of the control problem have been considered. Some numerical simulations are given to validate the theoretical results.

 ISBN: XXX-XXX-XXXX-X-X

# Stability analysis of multi-valued neural networks with time delays

R. Rakkiyappan[1]

[1] Department of Mathematics, Bharathiar University, Tamilnadu, India
Email: rakkigru@gmail.com

**Abstract**

Neural networks with values in multidimensional domains have been intensively studied over the last few years, because of their immense potentials in application perspective. For a complex-valued neural networks(CVNNs), the main challenge is the choice of activation functions. In a real-valued neural networks(RVNNs), the node activation functions is usually chosen to be a smooth, bounded and non constant function. There is no problem in selecting a real valued functions that satisfies these type of requirements. In a CVNNs any regular analytic function cannot be bounded unless it reduces to a constant. That is to say, the activations in CVNNs cannot be both bounded and analytic. Therefore, the activations are main challenge for CVNNs. Quaternion does not meet commutative law with respect to multiplication, so the quaternion is much harder than that of other networks like RVNNs, CVNNs. Quaternion-valued neural networks (QVNNs) generic extension of RVNNS or CVNNs. More recently, the octonion-valued neural networks(OVNNs) gains its attention since the octonian algebra is a different generalization of the complex and quaternion algebra with an 8-dimensional normed division algebra that can be defined over the field of real numbers. To avoid the non associativity of the octonions and the non commutativity of the quaternion, the OVNNs model is decomposed into four complex-valued systems, using the Cayley-Dickson construction method. The existence of time delays frequently causes oscilation, divergence in neural networks. Since the time delay dependent results are looser than the time delay independent ones when the delay are small. So it is necessary to propose the time delay dependent stability results. For different classes of activation functions, different approaches are necessary to analyze the relevant neural networks. Some new sufficient delay-dependent stability criteria are established to ensure the stability of equilibrium point for the addressed neural networks by constructing an appropriate Lyapunov-Krasovskii functionals(LKFs). The obtained stability criteria are established in the form of LMIs. Finally, numerical simulation results are provided to illustrate the validity of the theoretical results.

 ISBN: XXX-XXX-XXXX-X-X

# Characteristic functions of oscillatory functional differential equations of mixed type

Md.Abdul Malique[1]

[1]Department of Mathematics, University of Hafr Al Batin, KSA
Email: amalique@uohb.edu.sa

**Abstract**

In this paper we study characteristic functions of linear homogeneous functional differential equations of mixed type. Our interest is in the qualitative properties of characteristics functions of functional differential equations of mixed type. The motivation of the work is to study oscillatory and non-oscillatory solutions of functional differential equations. Characteristic function is a key tool of an oscillatory or non-oscillatory solution. We shall consider a basic differential equations of mixed type of the form

$$y'(t) = \sum_{k=-1,j=1}^{1,3} b_j y(t+k),$$

where $y(t) \in \mathcal{R}$, $k \in \mathcal{R}$, $k \in [-1,1]$, $b \in \mathcal{R}$, $b > 0$, $j > 0$, $j \in [1,3]$. We conclude with our study and further work.

# References

[1] T. Krisztin, (2000) Non-oscillation for functional differential equations of mixed type Journal of Mathematical Analysis and Applications, 245, pp 326–345.

[2] C.T.H Baker, G. Bocharov, E. Parmuzin, F.A. Rihan, Some aspects of causal & neutral equations used in modelling, *Journal of Computational and Applied Mathematics* **229** (2009) 335–349.

[3] J. M. Ferreira, N. J. Ford, Md. Abdul Malique, S. Pinelas, Y. Yan (2011), Analytical and numerical treatment of oscillatory mixed diiffrential equations with differentiable delays and advances, Journal of Computational and Applied Mathematics, 236, pp 5112 –5130.

 ISBN: XXX-XXX-XXXX-X-X

# Oscillation criteria for second-order quasi-linear neutral functional differential equation

Osama Mooaz[1] and Omr Bazighifan[2]

[1]Department of Mathematics, Faculty of Science, Mansoura University, Mansoura, 35516, Egypt
[2]Department of Mathematics, Faculty of Education, Hadhramout University, Yemen
Email: o_moaaz@mans.edu.eg

**Abstract**

In this work, new sufficient conditions for oscillation of second-order quasi-linear neutral functional differential equation are established. In numerous applications in biology, electrical engineering or physiology, the dependence on the past appears naturally. Differential equations with delay appears in modeling of these natural phenomena. One objective of our paper is to further simplify and complement some well-known results which were published recently in the literature. Firstly, we improve results of Liu et al. (2012) so that the condition guarantee that all solutions are oscillatory, and without imposing restrictions on the derivatives of $p(t)$, $\tau(t)$. As well, we simplify results of Saker [2010] and Wu et al. (2016) by obtaining criteria ensure oscillation without verifying the extra conditions. In order to support our results, we introduce illustrating example.

## References

[1] Liu, H., Meng, F., Liu, P.: Oscillation and asymptotic analysis on a new generalized Emden Fowler equation. Applied Mathematics and Computation, 219 (2012), 27392748.

[2] Saker, S.: Oscillation theory of delay di?erential and di?erence equations. VDM Verlag Dr. Muller, Saarbrucken (2010).

[3] Wu, Y., Yu, Y., Zhang, J., Xiao, J.: Oscillation criteria for second order Emden-Fowler functional di?erential equations of neutral type. Journal of Inequalities and Applications, (2016) 2016:328

 ISBN: XXX-XXX-XXXX-X-X

# Qualitative analysis of delayed SIR epidemic model with a saturated incidence rate

Mohamed-Naim Y. Anwar[1], Hebatallah J Alsakaji[2], Fathalla A. Rihan[2]

[1] Faculty of Engineering, Pharos University in Alexandria, Egypt

[2]Department of Mathematical Sciences, College of Science, UAE University, 15551, Al Ain, UAE

Email: mohamed.naeem@pua.edu.eg

**Abstract**

This work is concerned a delayed SIR epidemic model in which the susceptibles are assumed to satisfy the logistic equation and the incidence term is of saturated form with the susceptible. We investigate the qualitative behaviour of the model and find the conditions that guarantee the asymptotic stability of corresponding steady states. We present the conditions in the time lag in which the DDE model is stable. Hopf bifurcation analysis is also addressed. Numerical simulations are provided in order to illustrate the theoretical results and gain further insight into the behaviour of this system.

 ISBN: XXX-XXX-XXXX-X-X

# Partial differential equations with delay term

M.Yebdri[1]* and H. Harraga[1]

[1] Department of Mathematics, University of Tlemcenn, Tlemcen, Algeria

m_yebdri@mail.univ-tlemcen.dz (* Corresponding author: Mustapha Yebdri)

**Abstract**

The theory of differential equations with delay occupies an important place is the development of the theory of infinite dimensional dynamical systems. This theory covers ordinary and partial differential equations with delay([4, 9]). Comparing to functional differential equations, there are a few works dealing with partial differential equations with delay(see[6, 7, 8, 9]). Even if the theory of functional analysis (see [1, 2]) allow us to study a wide class of partial differential equations, the literature sources of fundamental facts and approaches for partial differential equations with delay term are very few [1, 2, 3].

In this talk we establish the existence of weak solutions to a class of partial differential equation with delay which involve a nonlinear delay. For this one proceed by the compacity method i.e. we construct approximate solutions by reduction to the finite dimensional, for example by Galerkin method. So we obtain the existence of approximate solutions using an existence theorem of solution of a system of ordinary differential equations. After, one go to the limit on the dimension (see [5]).

# References

[1] N.V. Azbelev, V.P. Maksimov and L.F. Rakhmatullina, *Introduction to the theory of functional differential equations,* Moscow, Nauka, 1991 .

[2] H.Brezis, *Analyse fonctionnelle : théorie et Application,* Université Pière et Marie Curie et Ecole Polytechnique, $2^{ieme}$ édition, MASSON Paris New York Barcelone Milan Mexico 1987.

[3] I. D. Chueshov, *Introduction to the Theory of Infinite-Dimensional Dissipative Systems,* Acta, Kharkov (1999) , (in Russian). English transl, Acta, Kharkov (2002) .

[4] J. K. Hale, *Theory of Functional Differential Equations,* Springer, Berlin-Heidelberg- New York, 1977.

[5] J. L. Lions, *Quelques Méthodes de Résolution des Problèmes aux Limites Non Linéaires,* Dunod, Paris, 1969 .

[6] A.V. Rezounenko, *Partial differential equations with discrete and distributed state-dependent delays,* Journal of Mathematical Analysis and Applications, 2007. Vol. $326$, Issue $2$, ($15$ February 2007), $1031 - 1045$. ( see preprint version : A.V. Rezounenko, Two models of partial differential equations with discrete and distributed state-dependent delays, preprint. March 22, 2005, http://arxiv.org/abs/math.DS/0503470 ).

[7] A. V. Rezounenko, *On a class of PDEs with nonlinear distributed in space and time state-dependent delay term,* Department of Mechanics and Mathematics, Kharkov University,4, Svobody Sqr., Kharkov, 61077, Ukraine, $2008$.

[8] A.V. Rezounenko, J. Wu, *A non-local PDE model for population dynamics with state-selective delay: local theory and global attractors* Journal of Computational and Applied Mathematics, Vol. $190$, Issues $1 - 2, P.99 - 113$, $2006$.

[9] J. Wu, *Theory and Applications of Partial Functional Differential Equations,* Springer-Verlag, New York, $1996$ .

# Stability and numerical simulations for an SIR epidemic model with distributed delay and vaccination

A. ELAZZOUZI[1]*, A. LAMRANI ALAOUI[2] and A. M. TILIOUA[2]

[1] Department of MPI, University of Sidi Mouhamed Ben Abdellah, FP Taza, LSI Laboratory, Morocco

[2] Department of Mathematics, University of Moulay Ismaïl, FST Errachidia, M2I Laboratory, MAMCS Group, Morocco

abdelhai.elazzouzi@usmba.ac.ma (* Corresponding author: A. ELAZZOUZI)

**Abstract**

This work is concerned with the global dynamics of an SIR epidemic model with a wide class of generalized nonlinear incidence rate, distributed delay and vaccination. By constructing a Lyapunov functionals, we show that the disease free equilibrium state is globally asymptotically stable when the basic reproduction number $R_0$ is less than or equal to one and the disease endemic equilibrium is globally asymptotically stable when $R_0$ is greater than one. We provide a numerical simulations to illustrate the effect of vaccination on the model. It is shown, when the basic reproduction number for vaccination-free model $R_0 > 1$ that the disease of the SIR model proposed in [3] can be controlled but it can not be eradicated. Here, in this work, we give an example in which for certain values of the vaccination parameter p the disease will die out even if $R_0 > 1$. In other side, we compare numerical results of the distributed delay model and the discrete time delay model.

# References

[1] C. C. McCluskey, *Global stability for an SIR epidemic model with delay and nonlinear incidence,* Nonlinear Analysis: Real World Applications, vol. 11, 3106-3109, 2010.

[2] David Tudor, *A deterministic model for herpes infections in human and animal populations.* SIAM Rev., 32(1):136-139, 1990.

[3] Yukihiko Nakata, Yoichi Enatsu & Yoshiaki Muroya, *Global stability of sir epidemic models with a wide class of nonlinear incidence rates and distributed delays.* Discrete and Continuous Dynamical Systems-B, 15(1):61-74, 2011.

 ISBN: XXX-XXX-XXXX-X-X

# Effect of CTL and antibody immune responses on the virus dynamics with both virus-to-cell and cell-to-cell transmissions and Holling type-II

Ahmed M. Elaiw and Shafeek A. Ghaleb

[1] Department of Mathematics, Faculty of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia

E-mail: shafeekye2@gmail.com

**Abstract**

Mathematical modeling and analysis of viral infection models have attracted the interest of several researchers during the last decades. These works can help researchers for better understanding the viral dynamical behavior and providing new suggestions for clinical treatment. The basic viral infection model presented in [1] has focused on modeling the interaction between three main compartments, uninfected cells, infected cells and free virus particles. Cytotoxic T Lymphocyte (CTL) cells and antibodies play central role of controlling the viral progression. CTL cells attack and kill the infected cells. The B cell produces antibodies to neutralize the viruses. Wodarz [2] has presented a viral infection model with both CTL and antibody immune responses. The model presented in [2] is based on the assumption that the uninfected cells becomes infected by contacting with the virus (virus-to-cell transmission). However, it has been reported in [3] that the uninfected cells can also become infected when it contacts an infected cell (cell-to-cell transmission).

In this paper, we propose and analyze the effect of CTL and antibody immune responses on the virus dynamics with both virus-to-cell and cell-to-cell transmissions. The infection rate is given by Holling-type incidence. We first show that the model is biologically acceptable by showing that the solutions of the model are nonnegative and bounded. We find the steady states of the model and investigate their global stability analysis. We derive five threshold parameters which fully determine the existence and stability of the five steady states of the model. The global stability of all steady states of the models is proven using Lyapunov method and applying LaSalle's invariance principle, we established the global asymptotic stability of the steady states of the model. To support our theoretical results we have performed some numerical simulations for the model. The results show the effect of the CTL and antibody immune response in controlling the disease and in removing the virus from the body.

 ISBN: XXX-XXX-XXXX-X-X

# References

[1] M.A. Nowak and C.R.M. Bangham, Population dynamics of immune responses to persistent viruses, Science, 272 (1996), 74-79.

[2] D. Wodarz, Hepatitis C virus dynamics and pathology: the role of CTL and antibody responses, Journal of General Virology, 84 (2003), 1743-1750.

[3] X. Lai, X. Zou, Modelling HIV-1 virus dynamics with both virus-to-cell infection and cell-to-cell transmission, SIAM Journal of Applied Mathematics, 74 (2014), 898?917.

 ISBN: XXX-XXX-XXXX-X-X

# Global stability of latent Chikungunya virus dynamics model with multitarget cells and discrete time delays

Ahmed M. Elaiw, Taofeek O. Alade and Saud M. Alsulami

[1] Department of Mathematics, Faculty of Science, King Abdulaziz University, P.O. Box 80203, Jeddah 21589, Saudi Arabia

E-mail: taofeekalade@gmail.com

**Abstract**

Chikungunya virus (CHIKV) is an alphavirus which is transmitted to humans by Aedes aegypti and Aedes albopictus mosquitoes. The CHIKV attacks the target cells and causes Chikungunya fever with symptoms such as rashes, headache and fever. Symptoms usually occur after an incubation period of 4-7 days. CHIKV has been an outbreak in the last decade in some parts of the world and has increased the population of people at risk [1]. Although CHIKV pathogenesis in humans is not yet completely understood, but recent outbreaks have provided deep understanding into the organs and cells involved in the viral replication. CHIKV infects and replicates in a variety of cells, such as fibroblasts, macrophages, monocytes, skeletal muscle satellite cells and other skin cells [2]. In the literature of CHIKV infection, most of the mathematical models presented described the disease transmission in mosquito and human populations [3-5]. This paper proposes a latent Chikungunya viral infection model with multitarget cells. The model incorporates (i) two types of infected cells, latently infected cells and actively infected cells which produce the CHIKV particles (ii) saturated incidence rate which is suitable to model the nonlinear dynamics of the CHIKV especially when its concentration is high (iii) antibody immune response. The model is an (3n+2)-dimensional system of nonlinear delay differential equations (DDEs) that describes the population dynamics of CHIKV, n categories of uninfected target cells, n categories of latently and actively infected cells. The model is incorporated by intracellular discrete time delays. The qualitative behavior of the model is studied. Using the method of Lyapunov function, we established the global stability of the equilibria of the model. The effect of the time delay on the stability of the equilibria has also been illustrated by numerical simulations.

**Keywords:** Global stability; Chikungunya virus infection; Time delays; Multitarget cells; Lyapunov function

# References

[1] Kristin M. Long, and Mark T. Heise, Protective and pathogenic responses to Chikungunya virus infection, Curr Trop Med Rep. 2015; 2(1): 13-21.

 ISBN: XXX-XXX-XXXX-X-X

[2] F. M. Lum, L. F. P. Ng, Cellular and molecular mechanisms of chikungunya pathogenesis, Antiviral Research 120 (2015), 165-174.

[3] X. Liu, and P. Stechlinski, Application of control strategies to a seasonal model of chikungunya disease, Applied Mathematical Modelling, 39 (2015), 3194-3220.

[4] A. M. Elaiw, T. O. Alade and S. M. Alsulami, Stability of a within-host Chikungunya virus dynamics model with latency, Journal of Computational Analysis and Applications, 26(5) 2019, 777-790.

[5] D. Moulay, M. Aziz-Alaoui, M.Cadivel, The chikungunya disease: modeling, vector and transmission global dynamics, Mathematical Biosciences, 229 (2011) 50-63.

 ISBN: XXX-XXX-XXXX-X-X

# Release of Breeding Suppression Advanced Time Delay Predator-Prey Dynamics

Qamar, J. A. Khan and Fatma A. Al-kharousi

Department of Mathematics, Sultan Qaboos University, Muscat, Oman

E-mail: qjalil@squ.edu.om

**Abstract**

This paper present a theoretical model where prey, due to short term heavy predation pressure, temporarily suppress breeding and restart breeding when in future predation pressure lowers. The predator consumes both suppressor and breeder of the prey and this prey population is more prone to predation at higher densities. Equilibrium and stability analysis are carried out. Taking fraction of new born breeder prey, q as a bifurcation parameter it is shown that Hopf bifurcation could occur.

**Keywords:** Prey-predator, Time delay model, Equilibrium points, Stability analysis, Hopf bifurcation

 ISBN: XXX-XXX-XXXX-X-X

# Dynamics of visceral leishmaniasis model with time delays

Velmurugan Gandhi, Ibrahim Elmojtaba and Nasser Al Salti

Department of Mathematics, Sultan Qaboos University, Muscat, Oman

E-mail: velmurugan@squ.edu.om

**Abstract**

In this work, authors discuss some of the dynamical characteristics of visceral leishmaniasis model with time delays. First, we propose a mathematical model for visceral leishmaniasis disease with time delay. Then, by analyzing the characteristic equations, some sufficient conditions are derived to ensure the stability of equilibrium points (disease-free equilibrium and endemic equilibrium) for the considered delayed visceral leishmaniasis model. Moreover, the time delay is taken as bifurcation parameter and establish some sufficient criteria to show the existence of Hopf bifurcation of the considered model. Finally, some numerical simulations are given to show the effectiveness of our theoretical results.

**Key words:** Endemic diseases, Leishmaniasis model with delay, Equilibrium points, Stability analysis, Bifurcation analysis, Numerical simulations.

 ISBN: XXX-XXX-XXXX-X-X

# SIR Model with Time-Varying Contact Rate

Fatma Al-Musalhi, Nasser Al-Salti, Ibrahim Elmojtaba, Velmurugan Gandhi

Department of Mathematics, Sultan Qaboos University, Muscat, Oman

E-mail: Fatma.almusalhi@outlook.com

**Abstract**

The contact rate is defined as the average number of contacts adequate for disease transmission by an individual per unit time and it is almost always assumed to be constant in time. However, in reality, the contact rate is not always constant throughout the year due to different factors such as population behaviour, environmental factors and many others. Therefore, it is more realistic to consider it to be a function of time.

In this talk, an SIR model with time-varying contact rate is considered. The existence and local stability of the equilibria of the model are analyzed. Results on global stability of disease-free equilibrium and transcritical bifurcation are proved. Finally, numerical simulations are presented to illustrate the theoretical results and to demonstrate the effect of the model parameters related to the contact rate on the behaviour of the solutions.

**Key words:** Endemic diseases, SIR model, Equilibrium points, Bifurcation analysis, Global stability, Numerical simulations.

 ISBN: XXX-XXX-XXXX-X-X

# Mathematical Analysis for the Dynamics of an Enteric Disease Model

Shaima Al Shanfari, Ibrahim M. Elmojtaba and Nasser Al Salti

Department of Mathematics, Sultan Qaboos University, Muscat, Oman

E-mail: s53774@student.squ.edu.om

**Abstract**

Enteric diseases range from mild food poisoning to dangerous infections like typhoid fever and cholera. Enteric and diarrheal diseases kill about 500,000 children under 5 each year, and those who survive face repeated infections by gut pathogens in the early years of their life, which can lead to serious, lifelong health problems. There are many challenges associated with modelling of enteric diseases. However, the most important are their indirect transmission mode and the role of mechanical vectors in the transmission circle. The aim of this research is to propose a new model to overcome those challenges. The model will be analyzed and all of its properties will be shown both analytically and numerically.

**Key words:** Enetric disease, Equilibrium points, Bifurcation analysis, Global stability, Optimal control, Numerical simulations

 ISBN: XXX-XXX-XXXX-X-X

# Prey-Predator Models with Variable carrying capacity

Maryam Al-Maqbali, Nasser Al-Salti and Ibrahim Elmojtaba

Department of Mathematics, Sultan Qaboos University, Muscat, Oman

E-mail: maryamalmoqbalix@gmail.com

**Abstract**

Prey-predator models with variable carrying capacity are proposed. These models are more realistic in modelling population dynamics in an environment that undergoes changes. In particular, prey-predator models with Holling type I and type II functional responses, incorporating the idea of a variable carrying capacity are considered. The carrying capacity was modelled by a logistic equation that increases sigmoidally between an initial value k0, i.e. the initial value of the carrying capacity, which lies between a lower value k1 and a final value k1 + k2, where k2 is the range on which the carrying capacity is changing. In order to examine the effect of the variable carrying capacity on the prey-predator dynamics, the two models were analyzed qualitatively using stability analysis and numerical solutions for the prey and the predator population densities were also obtained. Results on global stability and Hopf bifurcation of certain equilibrium points have been also presented. The effect of other model parameters on the prey-predator dynamics has been also examined. In particular, results on the effect of handling parameter and the predator?s death rate, which has been to be the bifurcation parameter were presented.

**Key words:** Prey-predator, variable carrying capacity, Limit cycle, Hopf bifurcation

 ISBN: XXX-XXX-XXXX-X-X

# Controllability analysis of nonlinear fractional order differential systems with state delay and non-instantaneous impulsive effects

B.Sundaravadivoo

Department of Mathematics, Alagappa University, Karaikudi-630 004, India

E-mail: sundaravadivoon@gmail.com

**Abstract**

This manuscript prospects the controllability criteria of non-instantaneous impulsive non-linear fractional differential systems with state delay. By enrolling an appropriate Grammian matrix which is often defined by the Mittag-Leffer function and with the assistance of Laplace transform, the conditions to obtain the necessary and sufficient for the controllability of non-instantaneous impulsive fractional differential systems with state delay are derived using algebraic approach. An important feature presents in the paper is that, taken non-instantaneous impulses into the fractional order dynamical system with state delay and studied the controllability analysis, since this not exists in the available source of literature. Inclusively, two illustrative examples are provided with the existence of state delay non-instantaneous impulse fractional dynamical system. So this demonstrates the validity and efficacy of the obtained criteria of the main section.

 ISBN: XXX-XXX-XXXX-X-X

# On the asymptotic stability and boundedness of solutions stochastic differential equations with multiple time lags

Cemil Tun and Osman Tun

Department of Mathematics, Alagappa University, Karaikudi-630 004, India

E-mails: cemtunc@yahoo.com & osmantunc89@gmail.com

**Abstract**

TIt this paper, we consider a class of stochastic non-linear differential equations of second order with multiple delays. We establish new sufficient conditions which guarantee stochastic stability, stochastic asymptotic stability of the zero solutions and boundedness of all solutions of the considered stochastic delay differential equations. The technique of the proof is based on the construction of suitable Lyapunov functionals. The results of this paper extend and improve some recent results that can be found literature on the subject, and they are new and have novelty. As an application of the obtained results, in the particular case, an example given for illustrations by using MATLAB-Simulink.

## References

[1] Abou-El-Ela, A.M., Sadek, A.R., Mahmoud, A.M., Farghaly, E.S., Asymptotic stability of solutions for a certain non-autonomous second-order stochastic delay differential equation. Turkish J. Math. 41 (2017), no.3, 576?584.

[2] Huang, Zaitang; Yang, Qi-Gui; Cao, Junfei, The stochastic stability and bifurcation behavior of an Internet congestion control model. Math. Comput. Modelling 54 (2011), no. 9-10, 1954?1965.

[3] Ladde, G. S.; Wu, Ling, Nonlinear stochastic differential equations: ARIMA models and applications. Proceedings of Neural, Parallel, and Scientific Computations. Volume 4, 236?240, Dynamic, Atlanta, GA, 2010.

[4] Lei, Jinzhi; Mackey, Michael C., Stochastic differential delay equation, moment stability, and application to hematopoietic stem cell regulation system. SIAM J. Appl. Math. 67 (2006/07), no. 2, 387?407.

## List of Participants:

 ISBN: XXX-XXX-XXXX-X-X