On properties of some graphs associated to groups

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Abstract

Let G be a group. The prime index graph of G, denoted by $\Pi(G)$, is an undirected graph whose vertices are all subgroups of G and two distinct comparable subgroups H and K are adjacent if and only if $[H : K]$ or $[K : H]$ is prime. In this talk, we investigate some properties of this graph, for example the connectivity of the prime index graph.

The Structure of Induced Simple Modules for 0-Hecke Algebras

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Abstract

In this talk we shall be concerned with the 0-Hecke algebra; its irreducible representations were classified and shown to be one-dimensional by Norton in 1979. The structure of a finite-dimensional module can be fully described by computing its submodule lattice. We will discuss how this can be encoded in a generally much smaller poset given certain conditions and state new branching rules in types B and D.

Hyperovals and bent functions

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Abstract

We consider Niho bent functions (they are equivalent to bent functions which are linear on the elements of a Desarguesian spread). We show that Niho bent functions are in one-to-one correspondence with line ovals in an affine plane. Furthermore, Niho bent functions are in one-to-one correspondence with ovals (in a projective plane) with nucleus at a fixed point. These connections allow us to present new simple descriptions of Subiaco and Adelaide hyperovals and their automorphism groups.

(m,n)-ideal elements and Ordered Semigroups

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Abstract

In this paper, we investigate $(0, m)$-ideal elements and 0-minimal $(0, m)$-ideal elements in *poe*-semigroups. Then, we define relations ${}_m\mathcal{I}, \mathcal{I}_n, \mathcal{B}_m^n, \mathcal{Q}_m^n$ and $\mathcal{H}_m^n$ on *le*-semigroups and prove that, on any *le*-semigroup, $\mathcal{B}_m^n \subseteq \mathcal{Q}_m^n \subseteq \mathcal{H}_m^n$. We also provide some sufficient conditions on *le*-semigroups under which these relations are equal to each other.

On special product-free sets in groups

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Abstract

A subset S of a group G is *product-free* if $ab \notin S$ for all $a, b \in S$. We call such a set '*locally maximal*' in G if it is not properly contained in any other product-free subset of G. These sets were first studied in 1974 by Street and Whitehead, who analysed some properties of locally maximal product-free sets, and introduced the concept of filled groups. In 1992, Clark and Petersen studied an application of locally maximal product-free sets in finite geometry, where they obtained an upper bound on the minimal sizes of locally maximal product-free sets in $\mathbb{Z}_2^n$ for $n \geq 4$. Giudici and Hart in a 2009 paper asked the question: which finite groups contain locally maximal product-free sets of given size k? They classified all finite groups that contain locally maximal product-free sets of sizes 1 and 2 and some of size 3, and conjectured that if a group G contains a locally maximal product-free sets of size 3, then $|G| \leq 24$. We give a proof of this conjecture. Moreover, we obtain partial results both for the size 4 and the general case, and discuss some open problems in this direction.

On cyclotomic polynomial coefficients

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Abstract

Recall that the n^{th}-cyclotomic polynomial Φ_n is defined by

$$\Phi_n(z) = \prod_{\zeta^n=1} (z - \zeta), \tag{1}$$

where ζ are the primitive roots of order n of the unity. One can easily check that the degree of Φ_n is $\varphi(n)$, where φ is the Euler totient function. Writing the polynomial $\Phi_n(z)$ in the algebraic form, one obtains

$$\Phi_n(z) = \sum_{j=0}^{\varphi(n)} c_j^{(n)} z^j, \tag{2}$$

where $c_j^{(n)}, j = 0, 1, \ldots, \varphi(n)$, are the coefficients of $\Phi_n(z)$. It is well known that all these coefficients are integers, while every cyclotomic polynomial is irreducible over $\mathbb{Z}$ (see for example, [4], Theorem 1, page 195).

Numerous interesting properties of the cyclotomic polynomials and their coefficients have been discovered over more than a hundred years. First, the polynomials up to $n < 105$ only have 0, 1 and -1 as coefficients. In 1883, Mignotti pointed out that -2 first appears as the coefficient of z^7 of P_{105}, while P_n only has the coefficients 0 and ± 1, whenever n is a product of at most two distinct primes. Then, 2 first appears for $n = 165$, while all coefficients of P_n do not exceed 2 in absolute value for $n < 385$. Later, in 1895 Bang showed that for $n = pqr$ with $p < q < r$ odd primes, no coefficient of P_n is larger than $p-1$. An important breakthrough came in 1931, when I. Schur show that the coefficients cyclotomic polynomials can be arbitrarily large in absolute value. For the early history of these results see the 1936 paper of E. Lehmer [6]. In 1987, Suzuki [10] proves that in fact, any integer can be a coefficient of a cyclotomic polynomial of a certain degree.

In this paper we establish an integral formula for the coefficients of the cyclotomic polynomial, which allows an elegant proof for the reciprocity of coefficients. We then discuss integer sequences related to the cyclotomic polynomial coefficients, such as the number of non-zero coefficients of $\Phi_n(z)$, or the first occurrence of n or $-n$ as a coefficient.

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Decomposition of self-dual codes over the ring $\mathbb{F}_2 + u\mathbb{F}_2 + v\mathbb{F}_2 + uv\mathbb{F}_2$

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Abstract

In this paper we first discuss few properties of the non-chain Frobenius ring $\mathbb{F}_2 + u\mathbb{F}_2 + v\mathbb{F}_2 + uv\mathbb{F}_2$, $u^2 = 0, v^2 = 0, uv = vu$. We discuss the decomposition of self-dual codes over the given ring $\mathbb{F}_2 + u\mathbb{F}_2 + v\mathbb{F}_2 + uv\mathbb{F}_2$ and see that the code can be decomposed into the direct sum of $C(\sigma)$ and $\mu(\sigma)$, for the permutation σ. We discuss few properties of self-dual codes over the ring and give the Decomposition theorem for self-dual codes. We define automorphism group of a code C and see how $Aut(C)$ and C are related with. In the end we define equivalent of codes and we discuss results connecting automorphism groups, equivalent codes and the decomposition of codes.

Algebraic construction of Near-bent functions via known power functions

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Abstract

Semi bent functions are 2- plateaued functions introduced by Chee, Lee and Kim in 1994. The study of semi bent functions has attracted the attention of many researchers due to their properties. In this paper we have given the algebraic construction of near bent functions defined over the finite field $\mathbb{F}_2^n$ (n odd) using the notion of trace function and Kasami and Gold power functions.

Coleman automorphisms of finite groups

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Abstract

A Coleman automorphism of a finite group G is an automorphism $\varphi \in \mathrm{Aut}(G)$ that is equal to an inner automorphism, when the domain is restricted to a Sylow subgroup of G. The study of Coleman automorphisms started due to their relevance for the Normalizer problem of integral group rings of finite groups. It can be shown that if a finite group G has the property that all its Coleman automorphisms are inner, the Normalizer problem holds. In their well-known paper "Coleman automorphisms of finite groups"[1] M. Hertweck and W. Kimmerle posed the question whether for several different classes of groups the group of Coleman automorphisms coincides with the group of inner automorphisms. They showed that for these classes of groups, this holds if we further assume the finite groups to be solvable. This talk will quickly cover the known results. The bulk of the talk, will be the new results that were reported on in my recently submitted paper [2]. These new results show that for several questions even more restrictions can be made. Moreover, we show that for several classes of finite groups, the group of Coleman automorphisms is precisely the group of inner automorphisms. This shows as an important corollary that the Normalizer problem has a positive answer for these classes of groups.

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Approximations in a Nearring using Set-Valued Homomorphisms

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Abstract

We give lower and upper rough set approximations of an interval valued L-fuzzy ideal of nearring N using the notion of strong set valued homomorphism. The lower and upper approximations depend on t-norms and t-conorms. We prove that the lower and upper approximations of a 3-prime ideal of N induced by a 3-strong set valued homomorphism are 3-prime ideals of N. Finally, we study rough set approximations using the notion of reference points.

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Reconstruction methods in Computed Tomography

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Abstract

The X-ray computed tomography (or CT scan) makes use of computer-processed combinations of many X-ray images taken from different angles to produce cross-sectional images (so called virtual "slices") of specific areas of a scanned object. It allows the user to see inside the object without cutting. Until today, the so-called Filtered Back Projection is the basic reconstruction method in Computed Tomography. This method based on the analytical inversion of the Radon transform using the Fourier central slice theorem.

The developing of Graphics Processing Unit (GPU) gave new possibilities to the reconstruction technologies in computed tomography. High speed GPUs enable increases in computing performance by exploiting the power of the GPU. This technical development inspired the developing of new reconstruction algorithms. A totally different approach of reconstruction is based on iterative statistical methods. This method needs more computing capacity, but the quality of the reconstructed image is higher. We show a new statistical based reconstruction algorithm.

On the number of indecomposable representations of given degree of a cyclic group over local rings of finite length

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Abstract

Matrix representations of finite groups over fields are studied well enough. In the classical case when the characteristic p of a field k does not divide the order of a group G (in particular, $p = 0$), it always has (up to equivalence) a finite number of indecomposable representations; moreover all indecomposable representations are irreducible and every of them is a direct summand of the regular representation. In the modular case when p divides the order of G, the group has a finite number of indecomposable representations if and only if it holds for its p-Sylow subgroup; for G to be a p-group this property holds only if it is cyclic. The problem of the classification of all indecomposable representations is considered in [1].

Let K denotes a commutative principal ideal local ring (having an unity) with nilpotent maximal ideal $R = tK \neq 0$ and let its characteristic be equal to p^s (p is simple, $s \geq 1$). For a finite group G of order $|G| > 1$, we denote by $\mathrm{ind}_K(G, n)$ the number of nonequivalent indecomposable matrix K-representations of degree n of G. From [2] it follows that $\mathrm{ind}_K(G, n) \geq |K/R|$ for any p-group G of order $|G| > 2$ and $n > 1$. We strengthen this result in the case of both cyclic groups and radicals.
(1) Let $K_0 = K/R$ and R be nilpotent of degree $m \geq 2$. Then, for any $n > 1$ and for a cyclic p-group G of some order N depending on n (hence of greater order), $\mathrm{ind}_K(G, n) \geq (n - 1)|K_0|$.
(2) Let the characteristic of K be p and $R = tK \neq 0$ with $t^2 = 0$. Then, for any cyclic p-group G and $n \geq |G|$, $\mathrm{ind}_K(G, n) \geq (|G| - 2)|K_0|$.

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Group algebra whose group of units is locally nilpotent

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Abstract

Let $V(FG)$ be the group of normalized units of the group algebra FG of a group G over the field F. Let $V_*(FG)$ be the unitary subgroup of the group $V(FG)$ under the classical involution $*$ of FG. An explicit list of groups G and rings K for which $V(KG)$ are nilpotent was fully obtained by I. Khripta (see [4, 5]). In [1] it was completely determined when $V(FG)$ is solvable. It is still a challenging problem whether $V(FG)$ is an Engel group. This question has a long history (see [2, 3, 6, 7]).

The Engel property of a group is close to its local nilpotency (see [8]), but these classes of groups do not coincide. A locally nilpotent group is always Engel.

We present a complete list of groups G and fields F for which:

(i) $V(FG)$ is a locally nilpotent group;

(ii) FG has a finite number of nilpotent elements and $V(FG)$ is Engel;

(iii) the group of unitary units $V_*(FG)$ is locally nilpotent.

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On a structure of a finitary linear group over a commutative ring

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Abstract

Let $FL_\nu(K)$ be a finitary linear group where K is a ring with the unit, ν is a linearly ordered set. $FL_\nu(K)$ is investigated in [1], [2]. In particular a finitary unitriangular group $UT_\nu(K)$ is studied in [2]. We investigated periodic subgroups of $FL_\nu(K)$ where K is a Dedekind ring [3] and K ia a commutative Noetherian ring [4].

In this paper we continue the study of a structure of a finitary linear group $FL_\nu(K)$ over a commutative ring K. Let $A = \bigoplus_{i=1}^{\nu} A_i$, $A_i \simeq K$. We consider A as an $FL_\nu(K)$-module.

The main result of this paper is the theorem.

Theorem. Let G be a finitely generated subgroup of $FL_\nu(K)$, K be a commutative ring. If $C_G(A) = 1$, then G has the series of normal subgroups $L \leq N \leq G$ such that L is abelian, N/L is locally nilpotent and hyperabelian, G/N is residually linear.

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Groups whose proper subgroups are (locally π−finite)-by-(locally nilpotent)

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Abstract

If $\mathfrak{X}$ is a class of groups, then a group G is called a *minimal non-$\mathfrak{X}$-group* if it is not an $\mathfrak{X}$-group but all its proper subgroups belong to $\mathfrak{X}$. Let π be a set of primes and let $\mathfrak{X}$ be a quotient and subgroup closed class of locally nilpotent groups such that every infinite locally graded minimal non-$\mathfrak{X}$-group is a countable p-group for some prime p. Our main result in the present paper states that G is an infinitely generated minimal non-$(L\mathfrak{F}_\pi)\mathfrak{X}$-group if and only if there exists a prime $p \notin \pi$ such that G is an infinitely generated minimal non-$\mathfrak{X}$ p-group; where $L\mathfrak{F}_\pi$ denotes the class of locally finite π-groups.

If $\mathfrak{X}$ is a class of groups, then a group G is called a *minimal non-$\mathfrak{X}$-group* if it is not an $\mathfrak{X}$-group but all its proper subgroups belong to $\mathfrak{X}$. Many results have been obtained on minimal non-$\mathfrak{X}$-groups for several choices of $\mathfrak{X}$. In particular, in [2] a complete description of infinitely generated minimal non-nilpotent groups having a maximal subgroup is given. These groups are metabelian Chernikov p-groups, where p is a prime. Later in [3], infinitely generated minimal non-nilpotent groups without maximal subgroups have been studied and it was proved, among many results, that they are countable p-groups. In [1] it is proved that if G is a minimal non-$(L\mathfrak{F})\mathfrak{N}$ (respectively, non-$(L\mathfrak{F})\mathfrak{N}_c$) group, then G is a finitely generated perfect group which has no proper subgroups of finite index and $G/Frat(G)$ is simple, where $L\mathfrak{F}$ (respectively, $\mathfrak{N}$, $\mathfrak{N}_c$) denotes the class of locally finite (respectively, nilpotent, nilpotent of class at most c) groups. Therefore there are no minimal non-$(L\mathfrak{F})\mathfrak{N}$-groups (respectively, non-$(L\mathfrak{F})\mathfrak{N}_c$-groups) which are infinitely generated (or equivalently locally graded). In the present paper, we generalize these last results by considering the classes $(L\mathfrak{F}_\pi)\mathfrak{N}$ and $(L\mathfrak{F}_\pi)\mathfrak{N}_c$, where π is a given set of primes and $L\mathfrak{F}_\pi$ denotes the class of locally finite π-groups. It turns out that infinitely generated minimal non-$(L\mathfrak{F}_\pi)\mathfrak{N}$-groups exist. For if G is an infinitely generated minimal non-nilpotent group, then it is a p-group for some prime p and hence it is an infinitely generated minimal non-$(L\mathfrak{F}_\pi)\mathfrak{N}$-group for every set π not containing p. We will prove that the converse is also true. In fact our results on $(L\mathfrak{F}_\pi)\mathfrak{N}$ and $(L\mathfrak{F}_\pi)\mathfrak{N}_c$ will be consequences of more general results on $(L\mathfrak{F}_\pi)\mathfrak{X}$ (respectively, $(L\mathfrak{F}_\pi)\mathfrak{V}$), where $\mathfrak{X}$ (respectively, $\mathfrak{V}$) denotes a quotient and subgroup closed class (respectively, a variety) of locally nilpotent groups such that infinite locally graded minimal non-$\mathfrak{X}$-groups are countable p-groups. Our main result is as follows.

Theorem 0.1. *A group G is an infinitely generated minimal non-$(L\mathfrak{F}_\pi)\mathfrak{X}$-group if and only if there exists a prime $p \notin \pi$ such that G is an infinitely generated minimal non-$\mathfrak{X}$ p-group.*

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IP*-sets in function field

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Abstract

The ring of polynomials over a finite field $F_p[x]$ has received much attention both from a combinatorial view point and in regard to its action on measurable dynamical system. In the case of $(\mathbb{Z},+)$ we know that the ideal generated by any nonzero element is an IP*-set. In the present article we first establish that the analogous result is true for $F_p[x]$. We further use this result to establish some mixing properties of the action of $(F_p[x],+)$. We shall also discuss on Khintchine's recurrence for the action of $(F_p[x]\setminus\{0\},\cdot)$ and we pose the following question

Question: Given any ergodic system $\left(X,\mathcal{B},\mu,T_{f\in(F_q[x],\cdot)}\right)$, are the sets,

$$\{f\in F_q[x]:\mu(A\cap T_fA\cap T_{f^2}A)>\mu(A)^3-\epsilon\}$$

and

$$\{f\in F_q\left[x\right]:\mu(A\cap T_fA\cap T_{f^2}A\cap T_{f^3}A>\mu(A)^4-\epsilon\}$$

syndetic subsets of $(F_q[x],\cdot)$?

Commutatively closed sets

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Abstract

A subset S of a ring R is commutatively closed (abbreviated CC) if for any elements $a, b \in R$, $ab \in S$ implies that $ba \in S$. Some natural subsets such as $U(R) - 1$, where $U(R)$ denotes the set of units of R or the set of nilpotent elements are easily checked to be CC. $\{1\}$ is CC iff R is Dedekind finite and $\{0\}$is CC iff R is reversible. In this talk we will give many more examples and we will relate this property to other classical ones such as clean or 2-primal rings. We introduce a natural equivalence relation related to this notion and exhibit some constructions of the classes.

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Complete intersection analogue of a Theorem of Bass

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Abstract

Let (R, m, k) be a commutative Noetherian ring. In the classical homological algebra, there exist two celebrated and important facts which are obtained by virtue of the (Peskine-Szpiro) intersection theorem as follow:

i) if there exists a nonzero R-module of finite injective dimension, then R is Cohen-Macaulay,

ii) if there exists a nonzero Cohen-Macaulay R-module of finite projective dimension, then R is Cohen-Macaulay.

Part (i) is known as a Theorem of Bass. In this lecture, we prove the complete intersection analogue of these facts by using Complete intersection homological dimensions.

On the Derived Length of Units in Group Algebra

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Abstract

The aim of this talk is to give a relation between the derived length of the group of units in a modular group algebra of a finite group over a field of strictly positive characteristic and the commutativity of the group. Let KG be the group algebra of a group G over a field K of positive characteristic p and let $U(KG) = U$ denote its multiplicative group of units. Shalev ([9]), Kurdics ([6]), Sahai and Chandra ([7],[8],[3],[4]) have investigated group algebras with units having derived length at most two and three respectively over fields of positive characteristic. From results of Baginski [1] and Balogh and Li [2], it followed that if G is a torsion nilpotent nonabelian group, then the derived length of U is at least $\lceil log_2(p+1)\rceil$. We extend the result from torsion nilpotent groups to finite groups G without any further condition on the group properties and with the unit group U having derived length at most four. We also prove commutativity of G when the derived length of U is smaller than $\lceil \log_2(2p)\rceil$ under certain additional hypothesis. Our results can be found in [5].

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Topological loops having Lie groups as multiplication groups

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Abstract

The multiplication group $Mult(L)$ of a topological loop L is the group topologically generated by all the left and right translations of L. Necessary and sufficient condition for a group G to be the multiplication group $Mult(L)$ of a loop L is that there are two special transversals A and B in G with respect to a subgroup H which results to be the stabilizer of the identity element of L in $Mult(L)$ (cf. [1]). Mostly, the group $Mult(L)$ for a topological loop L has infinite dimension. Here we give a precise description about the structure of finite-dimensional Lie groups which are multiplication groups of 3-dimensional topological loops. Moreover, we provide a list of quasi-simple, nilpotent and solvable Lie groups which occur as the group $Mult(L)$ of 3-dimensional topological loops L (cf. [2], [3], [4]).

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On Dynamical Behaviors of p-adic Ising-Vannimenus model on the Cayley tree of order three

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Abstract

In [1] we had studied phase transition of the p-adic Ising-Vannimenus model with the interactions of nearest and next-nearest neighbors on the Cayley tree of order two. And in [2] author had studied the phase transition and Gibbs measures for the model on the Cayley tree of order $k = 3$, in real setting. In this study we consider the dynamic behaviors of the fixed points of this model in p-adic case. Due to the dynamic equations which are found in [2] in real case, we look for the fixed points of the recurrence equation which proves the existence of the translation invariant p-adic Gibbs measures (TIpGB) for the p-adic Ising-Vannimenus model on the semi-finite Cayley tree of order three.

In the present work we proved that the recurrent equation has four non-trivial fixed points that one of the fixed points is in $\mathcal{E}_p$ and the others are in $\mathbb{Z}_p^\star$. And we investigated the dynamic behaviors of the fixed points of the model and we concluded that u_0 is attractive and the fixed points u_1, u_2, u_3 are repelling if $u_i = p - 1$, neutral if $u_i \neq p - 1$.

Keywords:p-adic numbers, Cayley tree, Dynamic Behavior, Gibbs measures, Ising-Vannimenus model.

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Derivations of Genetic Volterra Algebras

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Abstract

There exist several classes of non-associative algebras (baric, evolution, Bernstein, train, stochastic, etc.), whose investigation has provided a number of significant contributions to theoretical population genetics [6]. Such classes have been defined different times by several authors, and all algebras belonging to these classes are generally called *genetic*. In [2] it was introduced the formal language of abstract algebra to the study of the genetics. Note that problems of population genetics can be traced back to Bernstein's work [1] where evolution operators were studied. Such kind of operators are mostly described by quadratic stochastic operators (QSO). A quadratic stochastic operator is used to present the time evolution of species in biology [3]. In [5], it was given along self-contained exposition of the recent achievements and open problems in the theory of the QSO.

Note that each QSO defines an algebraic structure on the vector space R^m containing the simplex (see next section for definitions). Such an algebra is called *genetic algebra*. Several works are devoted (see [4]) to certain properties of these algebras. We point out that the algebras that arise in genetics (via gametic, zygotic, or copular algebras) have very interesting structures. They are generally commutative but nonassociative, yet they are not necessarily Lie, Jordan, or alternative algebras. In addition, many of the algebraic properties of these structures have genetic significance. Therefore, it is the interplay between the purely mathematical structure and the corresponding genetic properties that makes this subject so fascinating. In population genetics, it is important to study dynamics of so-called Volterra operators. However, genetic algebras associated to these operators were not completely studied yet. Therefore, in the present work, we are going systematically investigate these kind of algebras. Moreover, we fully describe associative genetic Volterra algebras, in the later case, all derivations are trivial. Furthermore, we consider a general setting, i.e. the algebra is not necessarily associative. In this case, we provide a sufficient condition to get a trivial derivation on generic Volterra algebra.

The present work has been done jointly by R. Ganikhodzhev, A. Pirnapasov, I. Qaralleh.

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On skew cyclic codes over a finite ring

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Abstract

Let θ_t be an automorphism on ring R. Then a linear code C of length n over R is called a skew cyclic code or θ_t-cyclic code if for each $c = (c_0, c_1, \cdots, c_{n-1}) \in C$ implies that $\sigma(c) = (\theta_t(c_{n-1}), \theta_t(c_0), \cdots, \theta_t(c_{n-2})) \in C$. In this paper, we study skew cyclic codes over the ring $F_q + uF_q + vF_q$, where $u^2 = u,\ v^2 = v, uv = vu = 0,\ q = p^m$ and p is a prime. We define a Gray map from $F_q + uF_q + vF_q$ to F_q^3 and investigate the structural properties of skew cyclic codes over $F_q + uF_q + vF_q$ using decomposition method. It is shown that the Gray images of skew cyclic codes of length n over $F_q + uF_q + vF_q$ are the skew 3-quasi cyclic codes of length $3n$ over F_q. Finally, the idempotent generators of skew cyclic codes over $F_q + uF_q + vF_q$ have also been discussed.

Invariant ring of Aut(V, H)

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Abstract

Let V be a finite dimensional vector space over the finite field F_q with basis $e_1, \ldots, e_n$. Suppose $x_1, \ldots, x_n$ is the dual basis of the dual vector space V^*. Let $G \leq GL(V)$ and consider the polynomial ring in the n indeterminates $F_q[x_1, \ldots, x_n]$. Invariant theory over finite fields is a branch of abstract algebra. The theory deals with those elements of $F_q[x_1, \ldots, x_n]$ which do not change under the action of the group G. These elements form a ring structure which is called the ring of invariants of the group G. In my talk, I will discuss the ring of invariants of the following subgroup of $GL(V)$.

$$\text{Aut}(V, H) = \{g \in GL(V) : H(gv_1, gv_2) = H(v_1, v_2) \; \forall \; v_1, v_2 \in V\}$$

where H is a singular hermitian form on V.

$(\epsilon, \epsilon \bigvee q_k)$ fuzzy Γ-subsemigroup

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Abstract

In this research, $(\epsilon, \epsilon \bigvee q_k)$ fuzzy Γ-subsemigroup is defined and some examples are given. This is the generalization of $(\epsilon, \epsilon \bigvee q_k)$ fuzzy subsemigroup. Also $(\epsilon, \epsilon \bigvee q_k)$ fuzzy Γ-ideals, $(\epsilon, \epsilon \bigvee q_k)$ fuzzy left (right) Γ-ideals, $(\epsilon, \epsilon \bigvee q_k)$ fuzzy bi Γ-ideals and $(\epsilon, \epsilon \bigvee q_k)$ fuzzy generalized bi Γ-ideals are defined in Γ semigroup and then some properties of these ideals are obtained.

On an example of maximal commutative subalgebras of Grassmann algebra

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Abstract

The Grassman Algebra $G(n)$ of a vector space of dimension n has dimension 2^n. In 2014, Domokos and Zubor conjectured that, if $n = 4k+1$, then any maximal commutative subalgebra in $G(n)$ has dimension greater than $3(2)^{n-2}$. For $n = 4k + 1$ and $13 < n < 1000$, we give an explicit construction of maximal commutative subalgebras which have dimension less than $3(2)^{n-2}$. It serves as a counterexample to the conjecture raised by Domokos and Zubor.

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Ring Extensions with a finite number of intermediate rings

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Abstract

Let $R \subset S$ be an extension of integral domains. If T is a subring of S, we assume that T has the same unity element of S. The set of subrings of S that contain R is called the set of intermediate rings in the ring extension $R \subset S$. We let $[R, S]$ denote this set. If K is the field of fractions of R, then an intermediate ring in the ring extension $R \subset K$ is called an overring of R. If each overring of R is integrally closed in K, then R is called a Pruefer domain. There has been recently an increasing interest in ring extensions with only finitely many intermediate rings $[R, S]$, and in integral domains that have only finitely many overrings. Necessary and sufficient conditions for the finiteness of the number of intermediate rings in such ring extensions have been established by several authors. Several approximations for the number of intermediate rings in these ring extensions have been recently obtained, however the exact value of this number has been only computed in some special cases. The purpose of this paper is to present some related new results and highlight several open problems.

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Contributions to generalized derivation on prime near-ring with its application in the prime graph

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Abstract

In this talk, we discuss the notion of prime near- ring, which was introduced by Bell and Mason (In near-rings and near-fields North Holland Math. Studies, $137(1987), 31-35$), and Wang (Proc.Amer.Math.Soc., $121(1994), 361-366$) independently. Recently, many authors have investigated commutativity of prime and semi-prime rings admitting suitably constrained derivations. In 1992, Daif and Bell showed that a prime ring R must be commutative if it admits a derivation d such that either $d([x,y]) = [x,y]$ or $d([x,y]) = -[x,y]$ $\forall x, y \in I$, where I is a nonzero ideal of R. Note that the zero divisor graph of a commutative ring R is a graph with the set of non-zero zero divisors of R as the vertices and any two vertices x, y are adjacent if and only if $x \neq y$ and $xy = 0$. Some comparable results on near-rings have also been derived by several mathematician. The prime graph of a near-ring is a graph with vertices as the set of elements of N and edges as the set of vertex pair x, y such that $xNy = 0$. Indeed N is prime if and only if prime graph is a star graph (Comm. Algebra, 38(2010), 1957-1967). The objective of this paper is to extend some results on prime rings admitting generalized derivation to prime near-rings, and some results on relationship between the prime graph and the zero-divisor graph of N. In addition, examples are given to demonstrate the primeness in the hypothesis is not superfluous. Finally, we pose some problems.

Generalizations of Roth's criteria for solvability of matrix equations

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Abstract

The matrix equation $AX - XB = C$ has a solution if and only if the matrices $\left[\begin{smallmatrix} A & C \\ 0 & B \end{smallmatrix}\right]$ and $\left[\begin{smallmatrix} A & 0 \\ 0 & B \end{smallmatrix}\right]$ are similar. This criterion was proved over a field by W.E. Roth (1952) and over the skew field of quaternions by H. Liping (2001).

Dmytryshyn and Kågström [2, Theorem 6.1] extended Roth's criteria to the system of generalized Sylvester equations

$$A_i X_{i'} M_i - N_i X_{i''}^{\sigma_i} B_i = C_i, \qquad i = 1, \dots, s, \quad i', i'' = 1, \dots, t$$

over a field with involution, in which every $X_{i''}^{\sigma_i}$ is either $X_{i''}$, or $X_{i''}^T$, or $X_{i''}^*$. However, Dmytryshyn and Kågström [2] do not consider complex matrix equations that include the complex conjugate of unknown matrices.

Dmytryshyn and Kågström's criterion was extended in [1] to a large class of matrix equations that includes the systems

$$A_i X_{i'}^{\varepsilon_i} M_i - N_i X_{i''}^{\delta_i} B_i = C_i, \qquad i = 1, \dots, s, \quad i', i'' = 1, \dots, t$$

- of complex matrix equations, in which $\varepsilon_i, \delta_i \in \{1, \mathtt{C}, T, *\}$, where $X^{\mathtt{C}} := \bar{X}$ is the complex conjugate matrix and $X^* := \bar{X}^T$ is the complex adjoint matrix, and
- of quaternion matrix equations, in which $\varepsilon_i, \delta_i \in \{1, *\}$, where X^* is the quaternion adjoint matrix.

Roth criterion was extended in [3, 4] to the matrix equations $AX - \widehat{X}B = C$ and $X - A\widehat{X}B = C$ over the skew field of quaternions with an arbitrary involutive automorphism $q \mapsto \hat{q}$.

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On ideals of N-groups

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Abstract

Let N be a nearring and G a module over a nearrings (also called as N-group). We consider the notions i-uniform ideals ($i = 0, 1, 2, 3, 4$) in N-groups and obtained few results on N-group G, that has finite i-dimension in terms of i-uniform ideals. We also provide some examples to distinguish various uniform ideals of G. In case of modules over rings, some of these concepts will coincides.

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Modules over Infinite-Dimensional Algebras

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Abstract

Let A be an infinite dimensional K- algebra, where K is a field and let $\mathcal{B}$ be a basis for A. We explore when $K^{\mathcal{B}}$ (the direct product indexed by $\mathcal{B}$ of copies of the field K) can be made into an A-module in a natural way. We call a basis $\mathcal{B}$ satisfying that property "amenable," and we explore when amenable bases yield isomorphic A-modules. For the latter purpose, we consider a relation, which we name congeniality, that guarantees that two different bases yield (naturally) isomorphic A-module structures on $K^{\mathcal{B}}$. While amenability depends on the algebra structure, congeniality of bases depends only on the vector space structure and is thus independent from the specific algebra structure chosen. Among other results, we show that every algebra of countable infinite dimension has at least one amenable basis. Most of our examples will be within the familiar settings of the algebra $K[x]$ of polynomials with coefficients in K. We show that the relation of proper congeniality (when congeniality is not symmetric) yields several natural interesting questions; among these questions we highlight those related to a natural notion of simplicity of bases. We show that the algebra of polynomials with coefficients in K has at least as many truly distinct (so-called discordant) simple bases as there are elements in the base field K.

On different classes of Monomial Ideal associated to Lcm-lattices

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Abstract

Let K be a field and $S = K[x_1, x_2, ..., x_n]$ be a polynomial ring in n variables. To each monomial ideal I in S, we can compute its lcm-lattice $L(I)$. Let I and J are two monomial ideals such that their lcm-lattices are isomorphic. In this talk we will discuss different classes of ideals I and J such that $L(I^n)$ and $L(J^n)$ are isomorphic for different values of n. Further we will discuss algebraic and combinational properties of these ideals in terms of associated simplicial complexes.

Products of n homogeneous components in free Lie algebras

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Abstract

Let L be a free Lie algebra of finite rank $r \geq 2$ over a field F and we let L_{m_i} denote the degree m_i homogeneous component of L. Ralph Stöhr and Micheal Vaughan-Lee derived formulae for the dimension of the subspaces $[L_{m_1}, L_{m_2}]$ for all m_1 and m_2. The author and R. Stöhr obtained formulae for the dimension of the products $[L_{m_1}, L_{m_2}, L_{m_3}]$ under certain conditions on m_1, m_2, m_3. Then, Derya Karataş and the author obtained formulae for the dimension of the products of four homogeneous components. In this work, we investigated the products of n homogeneous components in free Lie algebra and we derived formulae for the dimension of such products.

Keywords: Free Lie algebras, homogeneous component.

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Bounded Engel elements in groups satisfying an identity

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Abstract

In this work, we proved that a residually finite group G satisfying an identity $w \equiv 1$ and generated by a commutator closed set X of bounded left Engel elements is locally nilpotent. We also extended such a result to locally graded groups, provided that X is a normal set. As an immediate consequence, we obtained that a locally graded group satisfying an identity, all of whose elements are bounded left Engel, is locally nilpotent.

Keywords: Engel element, residually finite group, restricted Burnside problem.

On Fuzzy Subgroups of a Certain Dihedral Group

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Abstract

In this work, an explicit formula for counting the number of distinct fuzzy subgroups of a certain dihedral group $D_{2P_1 \times P_2 \times \cdots P_n}$ is derived,(Where $P_1 \times P_2 \times \cdots P_n$ are distinct primes and n is any positive integer) with respect to a new equivalence relation $\approx$.

On 2-Banach algebras

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Abstract

The notion of linear 2- normed spaces was introduced by Siegfried Gahler in [5], which is nothing but a two dimensional analogue of a normed space. This concept had received the attention of a wider audience after the publication of a paper by A. G. White in [7]. In this paper, we introduce the idea of expansive, non-expansive and contraction mappings in linear 2-normed spaces eventually some of its properties are established. The analogue of Banach fixed point theorem for contraction mappings in linear 2- normed spaces is obtained. Some properties of resolvents in accretive operators are discussed. The concept of 2-Banach algebra with suitable examples and some related results are obtained.

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Derivation Of n-Dimensional Nilpotent Evolution Algebra

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Abstract

As a system of abstract algebra,evolution algebras are non associative algebras. There is no deep structure theorem for general non associative algebra. However, there are deep structure theorem and classification theorem for evolution algebras because it has been introduced concepts of dynamical systems to evolution algebras. In this work, we investigate the derivations of n-dimensional nilpotent evolution algebras, depending on the chooses of structure matrix. The spaces of derivations for nilpotent evolution algebras are described.

Application of Path Structure to the Conjugacy Classes of some Transformation Semigroup

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Abstract

The use of Path Structure (circuits and proper paths) was applied in the path decomposition of the conjugacy classes of some transformation semigroup. Conjugacy classes that are idempotent and nilpotent were also enumerated. Some properties(including general expression) were deduced from some of the conjugacy classes.

Wild Problems

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Abstract

Classification problems of representation theory are *wild* if they contain the problem of classifying matrix pairs up to similarity transformations

$$(A, B) \mapsto (S^{-1}AS, S^{-1}BS), \qquad S \text{ is nonsingular;}$$

the other problems are *tame*. These terms were introduced by P. Donovan and M.R. Freislich (1973) in analogy with the partition of animals into tame and wild. The problem of classifying matrix pairs up to similarity transformations contains the problem of classifying arbitrary systems of linear mappings (i.e., representations of an arbitrary quiver; see [3]), and so each wild problem is considered as hopeless.

I will talk about tame and wild classes of groups, associative algebras, Lie algebras, and systems of tensors. The talk is partly based on [1–4]. For example, let G be a finite p-group being a central extension of an abelian group B by an abelian group A:

$$1 \to A \to G \to B \to 1.$$

By [4], the class of all groups G with a fixed A is tame if and only if $|A| = p$, and the class of all groups G with a fixed B is tame if and only if $|B| = p$.

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Some Recent Results on Algebras Generated by Two Selfadjoint Idempotents

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Abstract

An algebra in question is isomorphic to a subalgebra of operators acting on a Hilbert space $\mathcal{H}$ and, as such, admits a canonical representation going back to [2]; see also [1] for further references.

In our talk we will show how this representation can be used to establish the exact formula for the distance from a given selfadjont idempotent to the set of selfadjoint idempotents orthogonal to their symmetries with respect to (also given) sefadjoint involution. This is an improvement of [5] obtained in [4].

Following [3], we will also provide the criterion for operators A from the algebra generated by two orthogonal projections to possess the compatible range property, i.e., coincide with A^* on the orthogonal complement to $\ker A + \ker A^*$.

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